3 Parallel and Perpendicular Lines

What You'll Learn

- **Lessons 3-1, 3-2, and 3-5** Identify angle relationships that occur with parallel lines and a transversal, and identify and prove lines parallel from given angle relationships.
- **Lessons 3-3 and 3-4** Use slope to analyze a line and to write its equation.
- **Lesson 3-6** Find the distance between a point and a line and between two parallel lines.

Key Vocabulary

- **parallel lines** (p. 126)
- **transversal** (p. 127)
- **slope** (p. 139)
- **equidistant** (p. 160)

Why It's Important

The framework of a wooden roller coaster is composed of millions of feet of intersecting lumber that often form parallel lines and transversals. Roller coaster designers, construction managers, and carpenters must know the relationships of angles created by parallel lines and their transversals to create a safe and stable ride. *You will find how measures of angles are used in carpentry and construction in Lesson 3-2.*

CONTENTS

Getting Started

▶ **Prerequisite Skills**   To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 3.
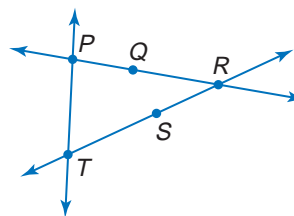
For Lesson 3-1   Naming Segments

Name all of the lines that contain the given point.

(For review, see Lesson 1-1.)

1. Q

2. R

3. S
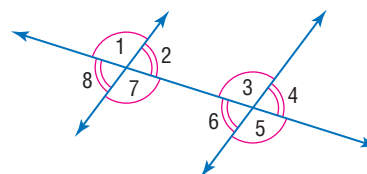
4. T

For Lessons 3-2 and 3-5   Congruent Angles

Name all angles congruent to the given angle.

(For review, see Lesson 1-4.)

5. $\angle 2$

6. $\angle 5$

7. $\angle 3$

8. $\angle 8$

For Lessons 3-3 and 3-4   Equations of Lines

For each equation, find the value of *y* for the given value of *x*.   *(For review, see pages 736 and 738.)*
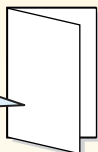
9. $y = 7x - 12$, for $x = 3$ **10.** $y = -\frac{2}{3}x + 4$, for $x = 8$ **11.** $2x - 4y = 18$, for $x = 6$

FOLDABLES™

Study Organizer

Parallel and Perpendicular Lines   Make this Foldable to help you organize information from this chapter. Begin with one sheet of $8\frac{1}{2}$-inch by 11-inch paper.
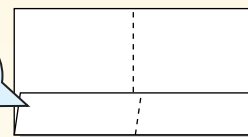
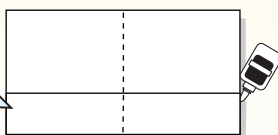**Step 1** **Fold**

Fold in half matching the short sides.

Step 2 **Fold Again**

Unfold and fold the long side up 2 inches to form a pocket.
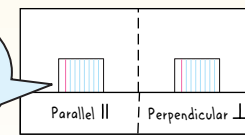
Step 3 **Staple or Glue**

Staple or glue the outer edges to complete the pocket.

Step 4 **Label**

Label each side as shown. Use index cards to record examples.

Parallel ∥   Perpendicular ⊥

Reading and Writing   As you read and study the chapter, write examples and notes about parallel and perpendicular lines on index cards. Place the cards in the appropriate pocket.

CONTENTS

# 3-1 Parallel Lines and Transversals

What You'll Learn

- Identify the relationships between two lines or two planes.
- Name angles formed by a pair of lines and a transversal.

How are parallel lines and planes used in architecture?

Architect Frank Lloyd Wright designed many buildings using basic shapes, lines, and planes. His building at the right has several examples of parallel lines, parallel planes, and skew lines.

Vocabulary

- parallel lines
- parallel planes
- skew lines
- transversal
- consecutive interior angles
- alternate exterior angles
- alternate interior angles
- corresponding angles
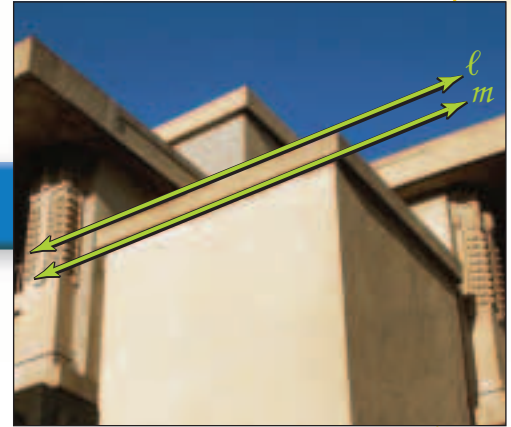
RELATIONSHIPS BETWEEN LINES AND PLANES Lines ℓ and m are coplanar because they lie in the same plane. If the lines were extended indefinitely, they would not intersect. Coplanar lines that do not intersect are called **parallel lines**. Segments and rays contained within parallel lines are also parallel.

The symbol ∥ means *is parallel to*. Arrows are used in diagrams to indicate that lines are parallel. In the figure, the arrows indicate that $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{RS}$.

$$\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$$
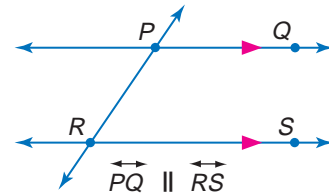
Similarly, two planes can intersect or be parallel. In the photograph above, the roofs of each level are contained in **parallel planes**. The walls and the floor of each level lie in intersecting planes.
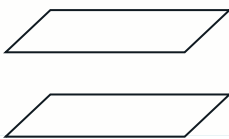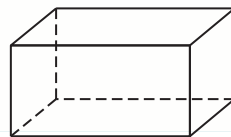
Geometry Activity

Draw a Rectangular Prism

A rectangular prism can be drawn using parallel lines and parallel planes.
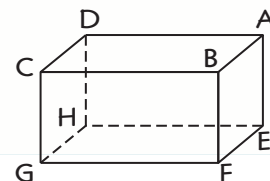
Step 1 Draw two parallel planes to represent the top and bottom of the prism.

Step 2 Draw the edges. Make any hidden edges of the prism dashed.

**Step 3** Label the vertices.

Analyze

1. Identify the parallel planes in the figure.
2. Name the planes that intersect plane *ABC* and name their intersections.
3. Identify all segments parallel to $\overline{BF}$.

Notice that in the Geometry Activity, $\overline{AE}$ and $\overline{GF}$ do not intersect. These segments are not parallel since they do not lie in the same plane. Lines that do not intersect and are not coplanar are called **skew lines**. Segments and rays contained in skew lines are also skew.

Example **1** **Identify Relationships**

a. **Name all planes that are parallel to plane ABG.**
   plane CDE

b. **Name all segments that intersect $\overline{CH}$.**
   $\overline{BC}$, $\overline{CD}$, $\overline{CE}$, $\overline{EH}$, and $\overline{GH}$

c. **Name all segments that are parallel to $\overline{EF}$.**
   $\overline{AD}$, $\overline{BC}$, and $\overline{GH}$

d. **Name all segments that are skew to $\overline{BG}$.**
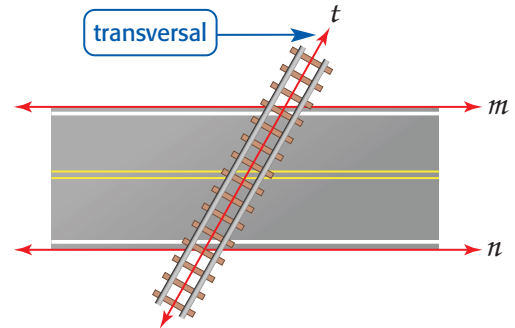 $\overline{AD}$, $\overline{CD}$, $\overline{CE}$, $\overline{EF}$, and $\overline{EH}$
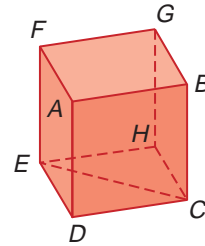
ANGLE RELATIONSHIPS In the drawing of the railroad crossing, notice that the tracks, represented by line t, intersect the sides of the road, represented by lines m and n. A line that intersects two or more lines in a plane at different points is called a **transversal**.
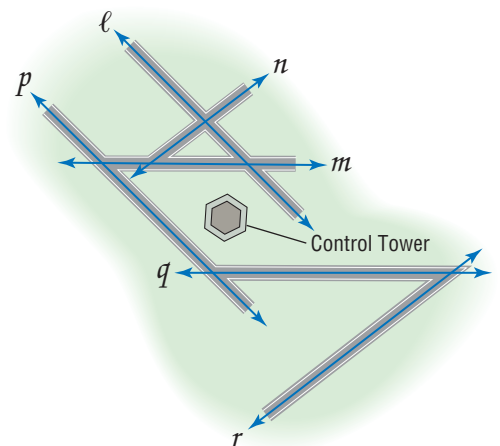
Example **2** **Identify Transversals**

AIRPORTS Some of the runways at O'Hare International Airport are shown below. Identify the sets of lines to which each given line is a transversal.

a. **line q**
   If the lines are extended, line q intersects lines ℓ, n, p, and r.

b. **line m**
   lines ℓ, n, p, and r

c. **line n**
   lines ℓ, m, p, and q

d. **line r**
   lines ℓ, m, p, and q

In the drawing of the railroad crossing above, notice that line t forms eight angles with lines m and n. These angles are given special names, as are specific pairings of these angles.
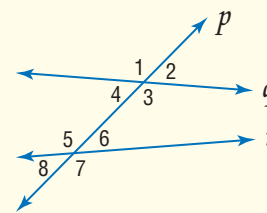
Study Tip

Same Side Interior Angles
Consecutive interior angles are also called *same side interior angles*.

Key Concept — Transversals and Angles

Name	Angles	
exterior angles	∠1, ∠2, ∠7, ∠8	Transversal p intersects lines q and r.
interior angles	∠3, ∠4, ∠5, ∠6	
consecutive interior angles	∠3 and ∠6, ∠4 and ∠5	
alternate exterior angles	∠1 and ∠7, ∠2 and ∠8	
alternate interior angles	∠3 and ∠5, ∠4 and ∠6	
corresponding angles	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8	

Example 3 — Identify Angle Relationships

Refer to the figure below. Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

a. ∠1 and ∠7
alternate exterior

b. ∠2 and ∠10
corresponding

c. ∠8 and ∠9
consecutive interior

d. ∠3 and ∠12
corresponding

e. ∠4 and ∠10
alternate interior

f. ∠6 and ∠11
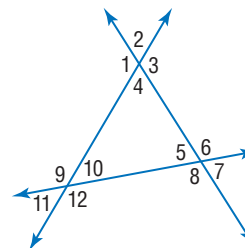alternate exterior
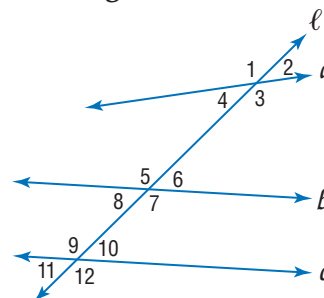
Check for Understanding

Concept Check

1. **OPEN ENDED** Draw a solid figure with parallel planes. Describe which parts of the figure are parallel.

2. **FIND THE ERROR** Juanita and Eric are naming alternate interior angles in the figure at the right. One of the angles must be ∠4.

Juanita
∠4 and ∠9
∠4 and ∠6

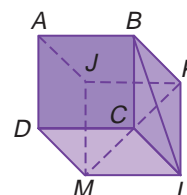Eric
∠4 and ∠10
∠4 and ∠5

Who is correct? Explain your reasoning.

3. **Describe** a real-life situation in which parallel lines seem to intersect.
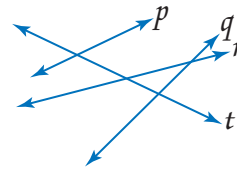
Guided Practice

For Exercises 4–6, refer to the figure at the right.

4. Name all planes that intersect plane *ADM*.
5. Name all segments that are parallel to $\overline{CD}$.
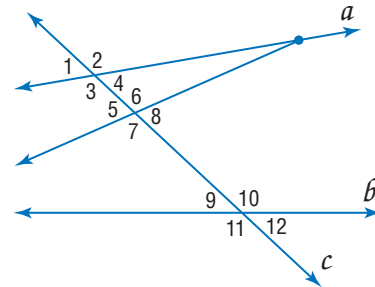6. Name all segments that intersect $\overline{KL}$.

CONTENTS

Identify the pairs of lines to which each given line is a transversal.
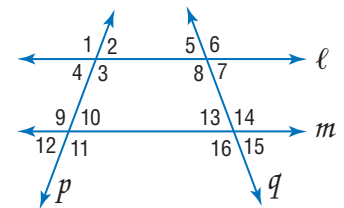
7. p

8. r

9. q

10. t

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

11. $\angle 7$ and $\angle 10$

12. $\angle 1$ and $\angle 5$

13. $\angle 4$ and $\angle 6$

14. $\angle 8$ and $\angle 1$

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair.

15. $\angle 3$ and $\angle 10$

16. $\angle 2$ and $\angle 12$

17. $\angle 8$ and $\angle 14$

Application   **MONUMENTS   For Exercises 18–21, refer to the photograph of the Lincoln Memorial.**

18. Describe a pair of parallel lines found on the Lincoln Memorial.

19. Find an example of parallel planes.

20. Locate a pair of skew lines.

21. Identify a transversal passing through a pair of lines.

Practice and Apply

Homework Help

For Exercises	See Examples
22–27	1
28–31	2
32–47	3

Extra Practice
See page 825.

For Exercises 22–27, refer to the figure at the right.

22. Name all segments parallel to $\overline{AB}$.

23. Name all planes intersecting plane BCR.

24. Name all segments parallel to $\overline{TU}$.

25. Name all segments skew to $\overline{DE}$.

26. Name all planes intersecting plane EDS.

27. Name all segments skew to $\overline{AP}$.
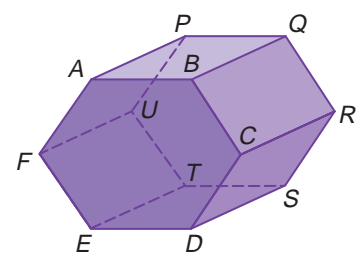
Identify the pairs of lines to which each given line is a transversal.

28. a

29. b

30. c

31. r
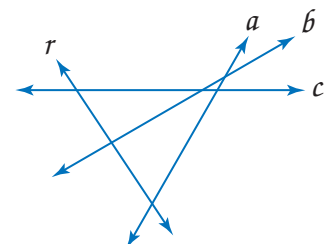
CONTENTS

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

32. ∠2 and ∠10 **33.** ∠1 and ∠11

34. ∠5 and ∠3 **35.** ∠6 and ∠14

36. ∠5 and ∠15 **37.** ∠11 and ∠13

38. ∠8 and ∠3 **39.** ∠9 and ∠4

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair.

40. ∠2 and ∠9 **41.** ∠7 and ∠15

42. ∠13 and ∠17 **43.** ∠8 and ∠4

44. ∠14 and ∠16 **45.** ∠6 and ∠14
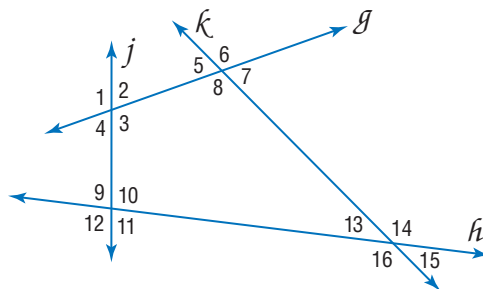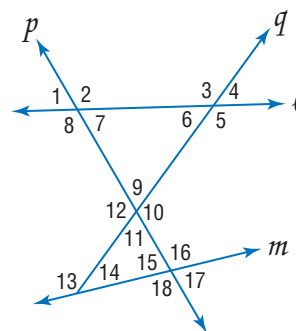
46. ∠8 and ∠6 **47.** ∠14 and ∠15

> **Study Tip**
>
> **Make a Sketch**
> Use patty paper or tracing paper to copy the figure. Use highlighters or colored pencils to identify the lines that compose each pair of angles.

48. AVIATION Airplanes heading eastbound are assigned an altitude level that is an odd number of thousands of feet. Airplanes heading westbound are assigned an altitude level that is an even number of thousands of feet. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning.

STRUCTURES For Exercises 49–51, refer to the drawing of the gazebo at the right.

49. Name all labeled segments parallel to $\overline{BF}$.

50. Name all labeled segments skew to $\overline{AC}$.

51. Are any of the planes on the gazebo parallel to plane *ADE*? Explain.

52. COMPUTERS The word *parallel* when used with computers describes processes that occur simultaneously, or devices, such as printers, that receive more than one bit of data at a time. Find two other examples for uses of the word *parallel* in other subject areas such as history, music, or sports.
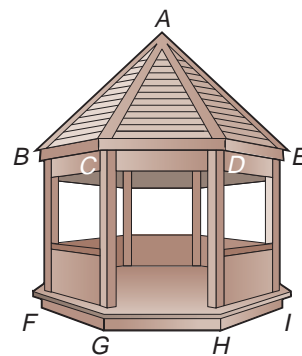
CRITICAL THINKING Suppose there is a line ℓ and a point *P* not on the line.

53. In space, how many lines can be drawn through *P* that do not intersect ℓ?

54. In space, how many lines can be drawn through *P* that are parallel to ℓ?

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.
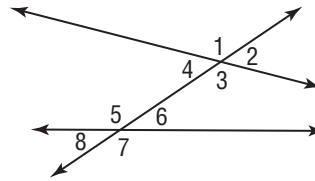
How are parallel lines and planes used in architecture?

Include the following in your answer:
- a description of where you might expect to find examples of parallel lines and parallel planes, and
- an example of skew lines and nonparallel planes.

CONTENTS

56. ∠3 and ∠5 are __?__ angles.

    Ⓐ alternate interior

    Ⓑ alternate exterior

    Ⓒ consecutive interior

    Ⓓ corresponding

57. GRID IN Set M consists of all multiples of 3 between 13 and 31. Set P consists of all multiples of 4 between 13 and 31. What is one possible number in P but NOT in M?
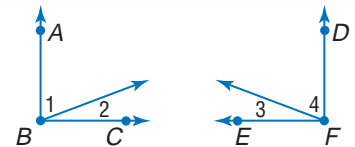
Maintain Your Skills

Mixed Review

58. PROOF Write a two-column proof. *(Lesson 2-8)*

    Given: $m\angle ABC = m\angle DFE, m\angle 1 = m\angle 4$

    Prove: $m\angle 2 = m\angle 3$

59. PROOF Write a paragraph proof. *(Lesson 2-7)*

    Given: $\overline{PQ} \cong \overline{ZY}, \overline{QR} \cong \overline{XY}$

    Prove: $\overline{PR} \cong \overline{XZ}$

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write *no conclusion*. *(Lesson 2-4)*

60. (1) If two angles are vertical, then they do not form a linear pair.

    (2) If two angles form a linear pair, then they are not congruent.

61. (1) If an angle is acute, then its measure is less than 90.

    (2) ∠*EFG* is acute.

Find the distance between each pair of points. *(Lesson 1-3)*

62. $A(-1, -8), B(3, 4)$ **63.** $C(0, 1), D(-2, 9)$ **64.** $E(-3, -12), F(5, 4)$

65. $G(4, -10), H(9, -25)$ **66.** $J\left(1, \frac{1}{4}\right), K\left(-3, -\frac{7}{4}\right)$ **67.** $L\left(-5, \frac{8}{5}\right), M\left(5, -\frac{2}{5}\right)$

Draw and label a figure for each relationship. *(Lesson 1-1)*

68. $\overleftrightarrow{AB}$ perpendicular to $\overleftrightarrow{MN}$ at point P

69. line ℓ contains R and S but not T

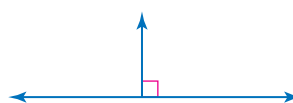**Getting Ready for the Next Lesson**

PREREQUISITE SKILL State the measures of linear pairs of angles in each figure. *(To review **linear pairs**, see Lesson 2-6.)*
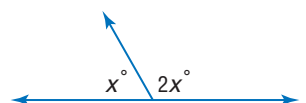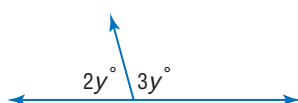
70.

50°

71.

72.

$x°$ $2x°$

73.

$2y°$ $3y°$

74.

$x°$ $2x°$ $3x°$

75.
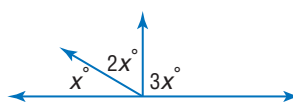
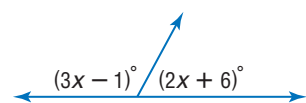$(3x - 1)°$ $(2x + 6)°$

CONTENTS

Geometry Software Investigation
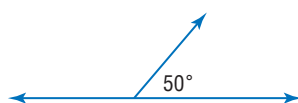
Angles and Parallel Lines

You can use The Geometer's Sketchpad to investigate the measures of angles formed by two parallel lines and a transversal.

Step 1 Draw parallel lines.
- Place two points *A* and *B* on the screen.
- Construct a line through the points.
- Place point *C* so that it does not lie on $\overleftrightarrow{AB}$.
- Construct a line through *C* parallel to $\overleftrightarrow{AB}$.
- Place point *D* on this line.
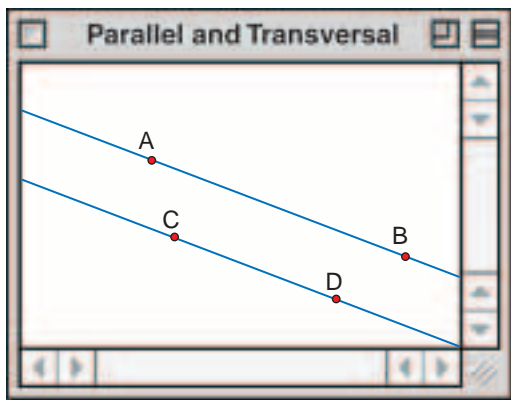
Step 2 Construct a transversal.
- Place point *E* on $\overleftrightarrow{AB}$ and point *F* on $\overleftrightarrow{CD}$.
- Construct $\overleftrightarrow{EF}$ as a transversal through $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.
- Place points *G* and *H* on $\overleftrightarrow{EF}$, as shown.

Step 3 Measure angles.
- Measure each angle.

Parallel and Transversal

Parallel and Transversal
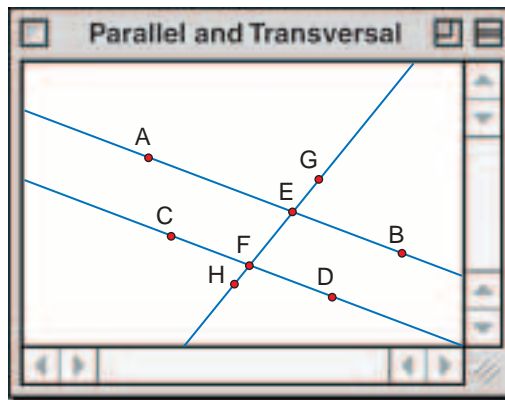
Analyze

1. List pairs of angles by the special names you learned in Lesson 3-1.
2. Which pairs of angles listed in Exercise 1 have the same measure?
3. What is the relationship between consecutive interior angles?

Make a Conjecture

4. Make a conjecture about the following pairs of angles formed by two parallel lines and a transversal. Write your conjecture in if-then form.
   a. corresponding angles
   b. alternate interior angles
   c. alternate exterior angles
   d. consecutive interior angles

5. Rotate the transversal. Are the angles with equal measures in the same relative location as the angles with equal measures in your original drawing?

6. Test your conjectures by rotating the transversal and analyzing the angles.

7. Rotate the transversal so that the measure of any of the angles is 90.
   a. What do you notice about the measures of the other angles?
   b. Make a conjecture about a transversal that is perpendicular to one of two parallel lines.

CONTENTS

3-2 Angles and Parallel Lines

What You'll Learn

- Use the properties of parallel lines to determine congruent angles.
- Use algebra to find angle measures.

How can angles and lines be used in art?

In the painting, the artist uses lines and transversals to create patterns. The figure on the painting shows two parallel lines with a transversal passing through them. There is a special relationship between the angle pairs formed by these lines.
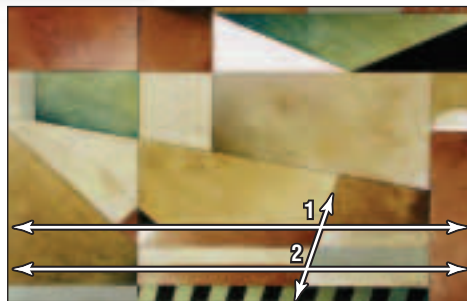
The Order of Tradition II by T.C. Stuart

PARALLEL LINES AND ANGLE PAIRS

In the figure above, $\angle 1$ and $\angle 2$ are corresponding angles. When the two lines are parallel, there is a special relationship between these pairs of angles.

Postulate 3.1

Corresponding Angles Postulate   If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Examples: $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, $\angle 4 \cong \angle 8$
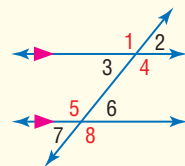
Example 1 Determine Angle Measures

In the figure, $m\angle 3 = 133$. Find $m\angle 5$.

$\angle 3 \cong \angle 7$	Corresponding Angles Postulate
$\angle 7 \cong \angle 5$	Vertical Angles Theorem
$\angle 3 \cong \angle 5$	Transitive Property
$m\angle 3 = m\angle 5$	Definition of congruent angles
$133 = m\angle 5$	Substitution
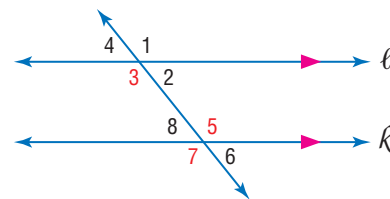
In Example 1, alternate interior angles 3 and 5 are congruent. This suggests another special relationship between angles formed by two parallel lines and a transversal. Other relationships are summarized in Theorems 3.1, 3.2, and 3.3.

CONTENTS

Key Concept — Parallel Lines and Angle Pairs

| Theorem	Examples
3.1 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.	$\angle 4 \cong \angle 5$ $\angle 3 \cong \angle 6$
3.2 Consecutive Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.	$m\angle 4 + m\angle 6 = 180$ $m\angle 3 + m\angle 5 = 180$
3.3 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.	$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$
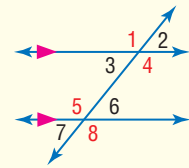
You will prove Theorems 3.2 and 3.3 in Exercises 40 and 39, respectively.

Proof Theorem 3.1

Given: $a \parallel b$; p is a transversal of a and b.

Prove: $\angle 2 \cong \angle 7, \angle 3 \cong \angle 6$

Paragraph Proof: We are given that $a \parallel b$ with a transversal p. By the Corresponding Angles Postulate, $\angle 2 \cong \angle 4$ and $\angle 8 \cong \angle 6$. Also, $\angle 4 \cong \angle 7$ and $\angle 3 \cong \angle 8$ because vertical angles are congruent. Therefore, $\angle 2 \cong \angle 7$ and $\angle 3 \cong \angle 6$ since congruence of angles is transitive.
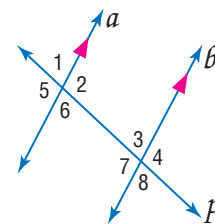
A special relationship occurs when the transversal is a perpendicular line.

Theorem 3.4

Perpendicular Transversal Theorem   In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.
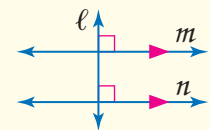
Proof Theorem 3.4

Given: $p \parallel q, t \perp p$

Prove: $t \perp q$

Proof:

Statements	Reasons
1. $p \parallel q, t \perp p$	**1.** Given
2. $\angle 1$ is a right angle.	**2.** Definition of $\perp$ lines
3. $m\angle 1 = 90$	**3.** Definition of right angle
4. $\angle 1 \cong \angle 2$	**4.** Corresponding Angles Postulate
5. $m\angle 1 = m\angle 2$	**5.** Definition of congruent angles
6. $m\angle 2 = 90$	**6.** Substitution Property
7. $\angle 2$ is a right angle.	**7.** Definition of right angles
8. $t \perp q$	**8.** Definition of $\perp$ lines

CONTENTS
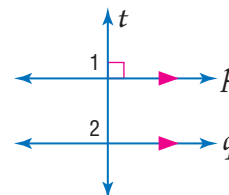
Example **2** **Use an Auxiliary Line**

Grid-In Test Item

What is the measure of $\angle GHI$?
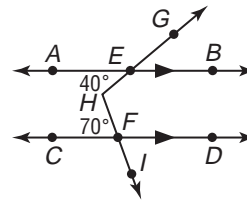
Read the Test Item

You need to find $m\angle GHI$. Be sure to identify it correctly on the figure.

Solve the Test Item

Draw $\overleftrightarrow{JK}$ through H parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

$\angle EHK \cong \angle AEH$	Alternate Interior Angles Theorem
$m\angle EHK = m\angle AEH$	Definition of congruent angles
$m\angle EHK = 40$	Substitution

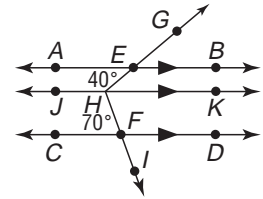|	
$\angle FHK \cong \angle CFH$	Alternate Interior Angles Theorem
$m\angle FHK = m\angle CFH$	Definition of congruent angles
$m\angle FHK = 70$	Substitution

$m\angle GHI = m\angle EHK + m\angle FHK$		Angle Addition Postulate
$= 40 + 70$ or 110		$m\angle EHK = 40, m\angle FHK = 70$

Write each digit of 110 in a column of the grid. Then shade in the corresponding bubble in each column.

The Princeton Review

Test-Taking Tip

Make a Drawing If you are allowed to write in your test booklet, sketch your drawings near the question to keep your work organized. Do not make any marks on the answer sheet except your answers.

ALGEBRA AND ANGLE MEASURES Angles formed by two parallel lines and a transversal can be used to find unknown values.

Example **3** **Find Values of Variables**

ALGEBRA If $m\angle 1 = 3x + 40$, $m\angle 2 = 2(y - 10)$, and $m\angle 3 = 2x + 70$, find x and y.

• Find x.

Since $\overrightarrow{FG} \parallel \overrightarrow{EH}$, $\angle 1 \cong \angle 3$ by the Corresponding Angles Postulate.

$m\angle 1 = m\angle 3$	Definition of congruent angles
$3x + 40 = 2x + 70$	Substitution
$x = 30$	Subtract 2x and 40 from each side.

• Find y.

Since $\overleftrightarrow{FE} \parallel \overleftrightarrow{GH}$, $\angle 1 \cong \angle 2$ by the Alternate Exterior Angles Theorem.

$m\angle 1 = m\angle 2$	Definition of congruent angles
$3x + 40 = 2(y - 10)$	Substitution
$3(30) + 40 = 2(y - 10)$	$x = 30$
$130 = 2y - 20$	Simplify.
$150 = 2y$	Add 20 to each side.
$75 = y$	Divide each side by 2.
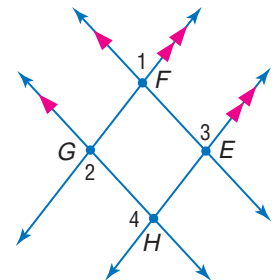
Concept Check

1. **Determine** whether ∠1 is *always*, *sometimes*, or *never* congruent to ∠2. Explain.

2. **OPEN ENDED** Use a straightedge and protractor to draw a pair of parallel lines cut by a transversal so that one pair of corresponding angles measures 35°.

3. **Determine** the minimum number of angle measures you would have to know to find the measures of all of the angles in the figure for Exercise 1.

4. **State** the postulate or theorem that allows you to conclude ∠3 ≅ ∠5 in the figure at the right.
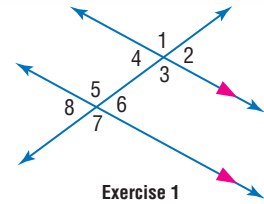
Exercise 1
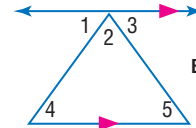
Exercise 4

Guided Practice

In the figure, $m\angle 3 = 110$ and $m\angle 12 = 55$. Find the measure of each angle.

5. ∠1   6. ∠6

7. ∠2   8. ∠10

9. ∠13   10. ∠15

Find x and y in each figure.

11.

$(8y + 2)°$ $(25y - 20)°$ $10x°$

12.

$(3y + 1)°$ $(4x - 5)°$ $(3x + 11)°$
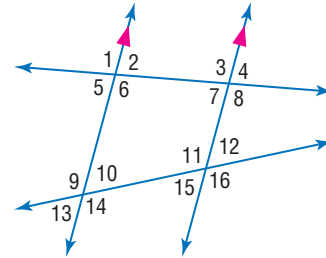
Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

13. **SHORT RESPONSE** Find $m\angle 1$.

36°   1   31°
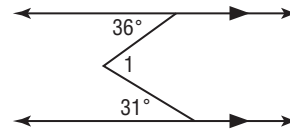
Homework Help

For Exercises	See Examples
14–31	1, 2
32–37	3

Extra Practice
See page 826.

In the figure, $m\angle 9 = 75$. Find the measure of each angle.

14. ∠3   15. ∠5

16. ∠6   17. ∠8

18. ∠11   19. ∠12
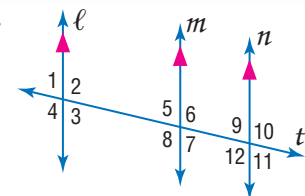
In the figure, $m\angle 3 = 43$. Find the measure of each angle.

20. ∠2   21. ∠7

22. ∠10   23. ∠11

24. ∠13   25. ∠16
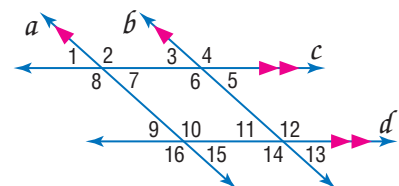
In the figure, $m\angle 1 = 50$ and $m\angle 3 = 60$. Find the measure of each angle.

26. ∠4   27. ∠5

28. ∠2   29. ∠6

30. ∠7   31. ∠8

CONTENTS
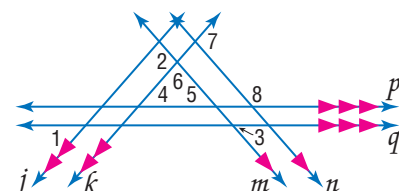
Find x and y in each figure.

32.
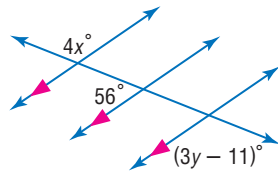
33.
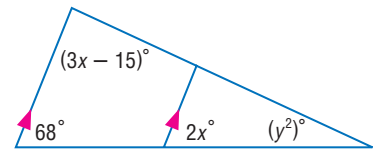
Find m∠1 in each figure.

34.
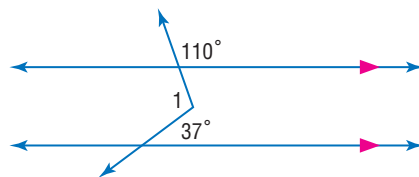
35.
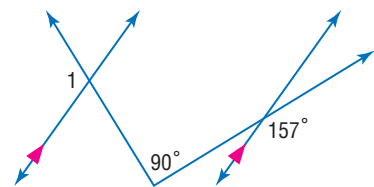
Find x, y, and z in each figure.

36.
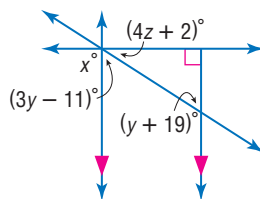
37.
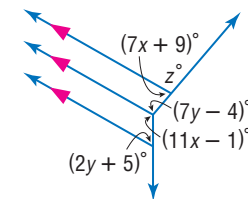
38. CARPENTRY Anthony is building a picnic table for his patio. He cut one of the legs at an angle of 40°. At what angle should he cut the other end to ensure that the top of the table is parallel to the ground?
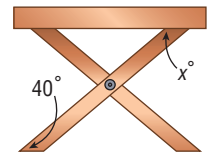
39. PROOF Copy and complete the proof of Theorem 3.3.

Given: $\ell \parallel m$

Prove: $\angle 1 \cong \angle 8$
$\angle 2 \cong \angle 7$

Proof:

Statements	Reasons
1. $\ell \parallel m$	1. ?
2. $\angle 1 \cong \angle 5, \angle 2 \cong \angle 6$	2. ?
3. $\angle 5 \cong \angle 8, \angle 6 \cong \angle 7$	3. ?
4. $\angle 1 \cong \angle 8, \angle 2 \cong \angle 7$	4. ?
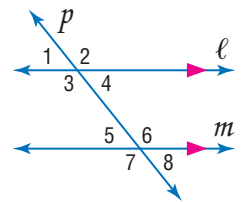
40. PROOF Write a two-column proof of Theorem 3.2.

41. PROOF Write a paragraph proof of Theorem 3.4.

42. CONSTRUCTION Parallel drainage pipes are laid on each side of Polaris Street. A pipe under the street connects the two pipes. The connector pipe makes a 65° angle as shown. What is the measure of the angle it makes with the pipe on the other side of the road?
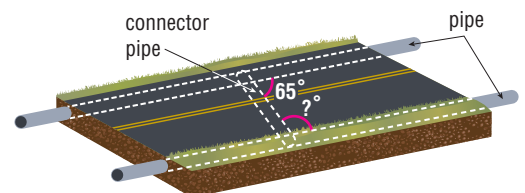
More About. . .

Construction

In 2001, the United States spent about $30 billion for federal highway projects.

Source: U.S. Dept. of Transportation

www.geometryonline.com/self_check_quiz

CONTENTS

Keith Wood/CORBIS

43. CRITICAL THINKING Explain why you can conclude that $\angle 2$ and $\angle 6$ are supplementary, but you cannot state that $\angle 4$ and $\angle 6$ are necessarily supplementary.
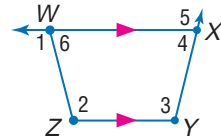
44. `WRITING IN MATH` Answer the question that was posed at the beginning of the lesson.

How can angles and lines be used in art?

Include the following in your answer:
- a description of how angles and lines are used to create patterns, and
- examples from two different artists that use lines and angles.

45. Line ℓ is parallel to line m. What is the value of x?
- Ⓐ 30
- Ⓑ 40
- Ⓒ 50
- Ⓓ 60
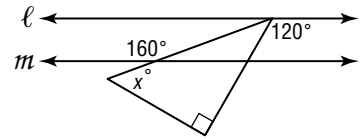
46. ALGEBRA If $ax = bx + c$, then what is the value of x in terms of a, b, and c?
- Ⓐ $\dfrac{c}{a+b}$
- Ⓑ $\dfrac{b}{a+c}$
- Ⓒ $\dfrac{c}{a-b}$
- Ⓓ $\dfrac{b+c}{a}$

Maintain Your Skills

Mixed Review

For Exercises 47–50, refer to the figure at the right. *(Lesson 3-1)*

47. Name all segments parallel to $\overline{AB}$.

48. Name all segments skew to $\overline{CH}$.

49. Name all planes parallel to AEF.

50. Name all segments intersecting $\overline{GH}$.
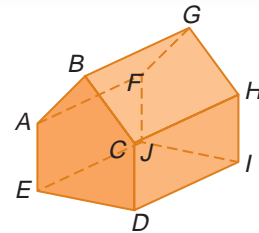
Find the measure of each numbered angle. *(Lesson 2-8)*

51.

52.
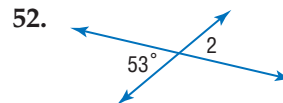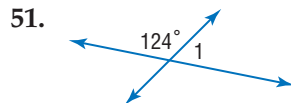
Identify the hypothesis and conclusion of each statement. *(Lesson 2-3)*

53. If it rains this evening, then I will mow the lawn tomorrow.

54. A balanced diet will keep you healthy.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each expression.
*(To review **simplifying expressions**, see pages 802 and 803.)*

55. $\dfrac{7-9}{8-5}$

56. $\dfrac{-3-6}{2-8}$

57. $\dfrac{14-11}{23-15}$

58. $\dfrac{15-23}{14-11}$

59. $\dfrac{2}{9} \cdot \left(-\dfrac{18}{5}\right)$

Practice Quiz 1
Lessons 3-1 and 3-2

State the transversal that forms each pair of angles. Then identify the special name for the angle pair. *(Lesson 3-1)*

1. $\angle 1$ and $\angle 8$

2. $\angle 6$ and $\angle 10$

3. $\angle 11$ and $\angle 14$

Find the measure of each angle if $\ell \parallel m$ and $m\angle 1 = 105$. *(Lesson 3-2)*

4. $\angle 6$

5. $\angle 4$

◀ **CONTENTS** ▶
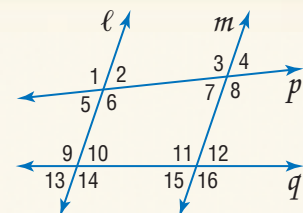
3-3 Slopes of Lines

- Find slopes of lines.
- Use slope to identify parallel and perpendicular lines.

Vocabulary

- slope
- rate of change

How is slope used in transportation?

Traffic signs are often used to alert drivers to road conditions. The sign at the right indicates a hill with a 6% *grade*. This means that the road will rise or fall 6 feet vertically for every 100 horizontal feet traveled.

6%

SLOPE OF A LINE
The **slope** of a line is the ratio of its vertical rise to its horizontal run.

$$\text{slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

In a coordinate plane, the slope of a line is the ratio of the change along the y-axis to the change along the x-axis.

vertical rise
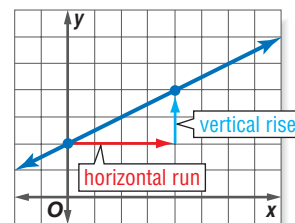
horizontal run

Key Concept
Slope

The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

The slope of a line indicates whether the line rises to the right, falls to the right, or is horizontal. The slope of a vertical line, where $x_1 = x_2$, is undefined.

Example 1 Find the Slope of a Line

Find the slope of each line.

a.

$(-1, 2)$

$(-3, -2)$

b.

$(-4, 0)$

$(0, -1)$

Study Tip

Slope

Lines with positive slope *rise* as you move from left to right, while lines with negative slope *fall* as you move from left to right.
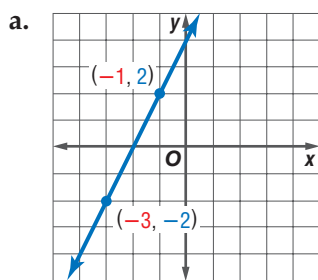
Use the $\frac{\text{rise}}{\text{run}}$ method.

From $(-3, -2)$ to $(-1, 2)$, go up 4 units and right 2 units.

$\frac{\text{rise}}{\text{run}} = \frac{4}{2}$ or 2
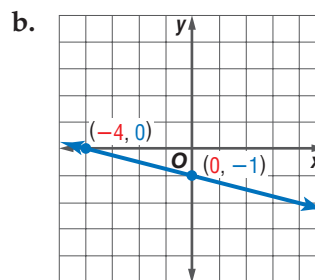
Use the slope formula.

Let $(-4, 0)$ be (x_1, y_1) and $(0, -1)$ be (x_2, y_2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1 - 0}{0 - (-4)}$ or $-\frac{1}{4}$

c.
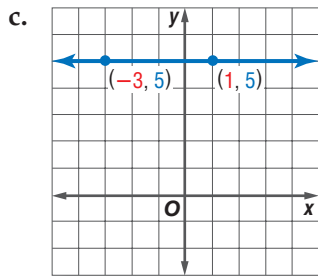
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 5}{-3 - 1}$$
$$= \frac{0}{-4} \text{ or } 0$$
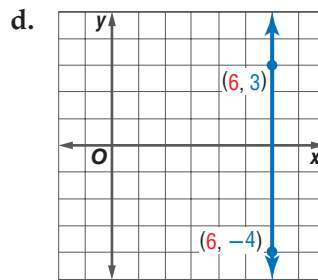
d.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3 - (-4)}{6 - 6}$$
$$= \frac{7}{0}, \text{ which is undefined}$$

The slope of a line can be used to identify the coordinates of any point on the line. It can also be used to describe a rate of change. The **rate of change** describes how a quantity is changing over time.

Example 2 Use Rate of Change to Solve a Problem

RECREATION Between 1990 and 2000, annual sales of inline skating equipment increased by an average rate of $92.4 million per year. In 2000, the total sales were $1074.4 million. If sales increase at the same rate, what will the total sales be in 2008?

Let $(x_1, y_1) = (2000, 1074.4)$ and $m = 92.4$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$92.4 = \frac{y_2 - 1074.4}{2008 - 2000} \qquad m = 92.4, y_1 = 1074.4, x_1 = 2000, \text{ and } x_2 = 2008$$

$$92.4 = \frac{y_2 - 1074.4}{8} \qquad \text{Simplify.}$$

$$739.2 = y_2 - 1074.4 \qquad \text{Multiply each side by 8.}$$

$$1813.6 = y_2 \qquad \text{Add 1074.4 to each side.}$$

The coordinates of the point representing the sales for 2008 are (2008, 1813.6). Thus, the total sales in 2008 will be about $1813.6 million.

PARALLEL AND PERPENDICULAR LINES

Examine the graphs of lines ℓ, m, and n. Lines ℓ and m are parallel, and n is perpendicular to ℓ and m. Let's investigate the slopes of these lines.

slope of ℓ	slope of m	slope of n
$m = \dfrac{2 - 5}{2 - (-3)}$	$m = \dfrac{1 - 4}{5 - 0}$	$m = \dfrac{2 - (-3)}{4 - 1}$
$= -\dfrac{3}{5}$	$= -\dfrac{3}{5}$	$= \dfrac{5}{3}$

Because lines ℓ and m are parallel, their slopes are the same. Line n is perpendicular to lines ℓ and m, and its slope is the opposite reciprocal of the slopes of ℓ and m; that is, $-\dfrac{3}{5} \cdot \dfrac{5}{3} = -1$. These results suggest two important algebraic properties of parallel and perpendicular lines.

CONTENTS
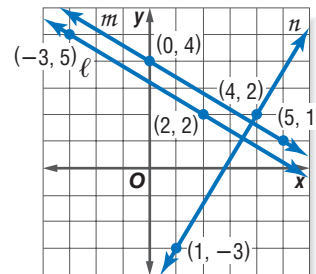
Study Tip

Look Back
To review **if and only if statements**, see Reading Mathematics, page 81.

<div style="border:1px solid">

Postulates   *Slopes of Parallel and Perpendicular Lines*

3.2   Two nonvertical lines have the same slope if and only if they are parallel.

3.3   Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

</div>

Example 3   Determine Line Relationships

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*.

a. $A(-2, -5)$, $B(4, 7)$, $C(0, 2)$, $D(8, -2)$

Find the slopes of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

$$\text{slope of } \overleftrightarrow{AB} = \frac{7 - (-5)}{4 - (-2)} \qquad\qquad \text{slope of } \overleftrightarrow{CD} = \frac{-2 - 2}{8 - 0}$$

$$= \frac{12}{6} \text{ or } 2 \qquad\qquad\qquad\qquad = -\frac{4}{8} \text{ or } -\frac{1}{2}$$

The product of the slopes is $2\left(-\frac{1}{2}\right)$ or -1. So, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

b. $A(-8, -7)$, $B(4, -4)$, $C(-2, -5)$, $D(1, 7)$

$$\text{slope of } \overleftrightarrow{AB} = \frac{-4 - (-7)}{4 - (-8)} \qquad\qquad \text{slope of } \overleftrightarrow{CD} = \frac{7 - (-5)}{1 - (-2)}$$

$$= \frac{3}{12} \text{ or } \frac{1}{4} \qquad\qquad\qquad\qquad = \frac{12}{3} \text{ or } 4$$

The slopes are not the same, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are not parallel. The product of the slopes is $4\left(\frac{1}{4}\right)$ or 1. So, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are neither parallel nor perpendicular.

Study Tip

Graphing Calculator
One way to make perpendicular lines appear perpendicular on a graphing calculator is to use the Zoom Square command.

   The relationships of the slopes of lines can be used to graph a line parallel or perpendicular to a given line.

Example 4   Use Slope to Graph a Line

Graph the line that contains $P(-2, 1)$ and is perpendicular to $\overleftrightarrow{JK}$ with $J(-5, -4)$ and $K(0, -2)$.

First, find the slope of $\overleftrightarrow{JK}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$= \frac{-2 - (-4)}{0 - (-5)} \qquad \text{Substitution}$$

$$= \frac{2}{5} \qquad \text{Simplify.}$$

The product of the slopes of two perpendicular lines is -1.

Since $\frac{2}{5}\left(-\frac{5}{2}\right) = -1$, the slope of the line perpendicular to $\overleftrightarrow{JK}$ through $P(-2, 1)$ is $-\frac{5}{2}$.

Graph the line. Start at $(-2, 1)$. Move down 5 units and then move right 2 units. Label the point Q. Draw $\overleftrightarrow{PQ}$.
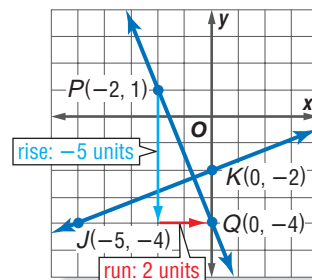
Study Tip

Negative Slopes
To help determine direction with negative slopes, remember that $-\frac{5}{2} = \frac{-5}{2} = \frac{5}{-2}$.

Check for Understanding

Concept Check

1. **Describe** what type of line is perpendicular to a vertical line. What type of line is parallel to a vertical line?

2. **FIND THE ERROR** Curtis and Lori calculated the slope of the line containing $A(15, 4)$ and $B(-6, -13)$.

Curtis
$$m = \frac{4 - (-13)}{15 - (-6)}$$
$$= \frac{17}{21}$$

Lori
$$m = \frac{4 - 13}{15 - 6}$$
$$= -\frac{9}{11}$$

Who is correct? Explain your reasoning.

3. **OPEN ENDED** Give an example of a line whose slope is 0 and an example of a line whose slope is undefined.

Guided Practice

4. Determine the slope of the line that contains $A(-4, 3)$ and $B(-2, -1)$.

Find the slope of each line.

5. ℓ

6. m

7. any line perpendicular to ℓ

Determine whether $\overleftrightarrow{GH}$ and $\overleftrightarrow{RS}$ are *parallel*, *perpendicular*, or *neither*.

8. $G(14, 13), H(-11, 0), R(-3, 7), S(-4, -5)$

9. $G(15, -9), H(9, -9), R(-4, -1), S(3, -1)$
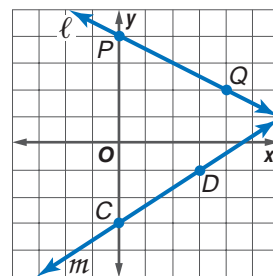
Exercises 5–7

Graph the line that satisfies each condition.

10. slope $= 2$, contains $P(1, 2)$

11. contains $A(6, 4)$, perpendicular to $\overleftrightarrow{MN}$ with $M(5, 0)$ and $N(1, 2)$

Application

MOUNTAIN BIKING For Exercises 12–14, use the following information.
A certain mountain bike trail has a section of trail with a grade of 8%.

12. What is the slope of the hill?

13. After riding on the trail, a biker is 120 meters below her original starting position. If her starting position is represented by the origin on a coordinate plane, what are the possible coordinates of her current position?

14. How far has she traveled down the hill? Round to the nearest meter.

Practice and Apply

Determine the slope of the line that contains the given points.

15. $A(0, 2), B(7, 3)$

16. $C(-2, -3), D(-6, -5)$

17. $W(3, 2), X(4, -3)$

18. $Y(1, 7), Z(4, 3)$

Determine whether $\overleftrightarrow{PQ}$ and $\overleftrightarrow{UV}$ are *parallel*, *perpendicular*, or *neither*.

19. $P(-3, -2), Q(9, 1), U(3, 6), V(5, -2)$

20. $P(-4, 0), Q(0, 3), U(-4, -3), V(8, 6)$

21. $P(-10, 7), Q(2, 1), U(4, 0), V(6, 1)$

22. $P(-9, 2), Q(0, 1), U(-1, 8), V(-2, -1)$

23. $P(1, 1), Q(9, 8), U(-6, 1), V(2, 8)$

24. $P(5, -4), Q(10, 0), U(9, -8), V(5, -13)$

CONTENTS

Find the slope of each line.

25. $\overleftrightarrow{AB}$

26. $\overleftrightarrow{PQ}$

27. $\overleftrightarrow{LM}$

28. $\overleftrightarrow{EF}$

29. a line parallel to $\overleftrightarrow{LM}$

30. a line perpendicular to $\overleftrightarrow{PQ}$

31. a line perpendicular to $\overleftrightarrow{EF}$

32. a line parallel to $\overleftrightarrow{AB}$
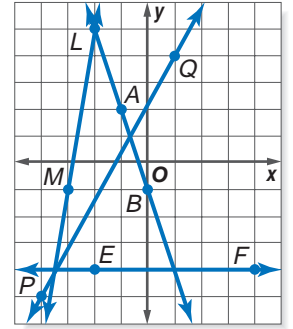
Graph the line that satisfies each condition.

33. slope $= -4$, passes through $P(-2, 1)$

34. contains $A(-1, -3)$, parallel to $\overleftrightarrow{CD}$ with $C(-1, 7)$ and $D(5, 1)$

35. contains $M(4, 1)$, perpendicular to $\overleftrightarrow{GH}$ with $G(0, 3)$ and $H(-3, 0)$

36. slope $= \frac{2}{5}$, contains $J(-7, -1)$

37. contains $Q(-2, -4)$, parallel to $\overleftrightarrow{KL}$ with $K(2, 7)$ and $L(2, -12)$

38. contains $W(6, 4)$, perpendicular to $\overleftrightarrow{DE}$ with $D(0, 2)$ and $E(5, 0)$.

USA TODAY Snapshots®

Median age continues to rise

The median age in the USA has more than doubled since 1820.

35.3

16.7

1820 1850 1900 1950 2000

Source: Census Bureau

By Sam Ward, USA TODAY
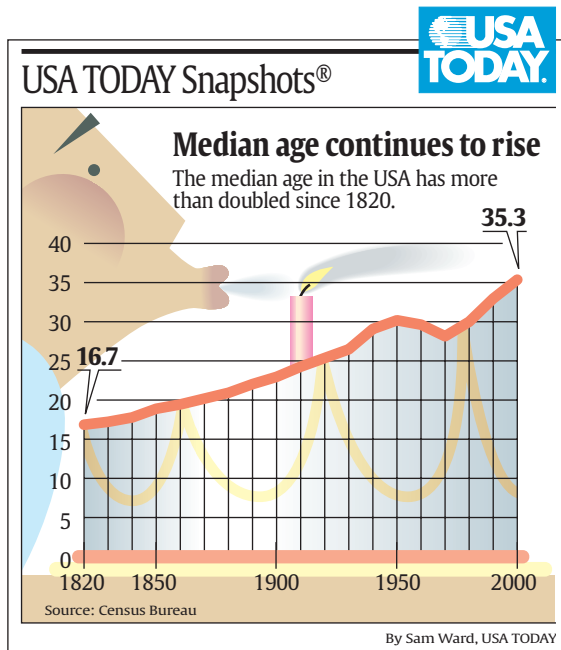
POPULATION For Exercises 39–41, refer to the graph.

39. Estimate the annual rate of change of the median age from 1970 to 2000.

40. If the median age continues to increase at the same rate, what will be the median age in 2010?

41. Suppose that after 2000, the median age increases by $\frac{1}{3}$ of a year anually. In what year will the median age be 40.6?

Online Research **Data Update** Use the Internet or other resource to find the median age in the United States for years after 2000. Does the median age increase at the same rate as it did in years leading up to 2000? Visit www.geometryonline.com/data_update to learn more.

42. Determine the value of x so that a line containing $(6, 2)$ and $(x, -1)$ has a slope of $-\frac{3}{7}$. Then graph the line.

43. Find the value of x so that the line containing $(4, 8)$ and $(2, -1)$ is perpendicular to the line containing $(x, 2)$ and $(-4, 5)$. Graph the lines.

COMPUTERS For Exercises 44–46, refer to the graph at the right.

44. What is the rate of change between 1998 and 2000?

45. If the percent of classrooms with Internet access increases at the same rate as it did between 1999 and 2000, in what year will 90% of classrooms have Internet access?

46. Will the graph continue to rise indefinitely? Explain.

Instructional Classrooms with Internet Access

Pecrent

64%

77%

51%

1998   1999   2000

Year
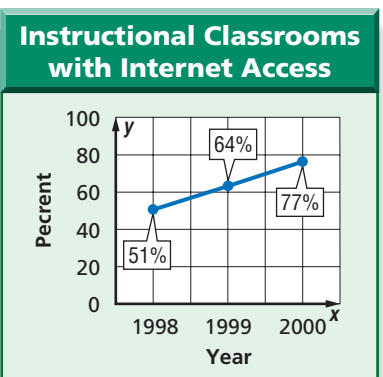
Source: U.S. Census Bureau

47. CRITICAL THINKING The line containing the point $(5 + 2t, -3 + t)$ can be described by the equations $x = 5 + 2t$ and $y = -3 + t$. Write the slope-intercept form of the equation of this line.

48. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How is slope used in transportation?

Include the following in your answer:
- an explanation of why it is important to display the grade of a road, and
- an example of slope used in transportation other than roads.

49. Find the slope of a line perpendicular to the line containing $(-5, 1)$ and $(-3, -2)$.

Ⓐ $-\dfrac{2}{3}$ Ⓑ $-\dfrac{3}{2}$ Ⓒ $\dfrac{2}{3}$ Ⓓ $\dfrac{3}{2}$

50. ALGEBRA The winning sailboat completed a 24-mile race at an average speed of 9 miles per hour. The second-place boat finished with an average speed of 8 miles per hour. How many minutes longer than the winner did the second-place boat take to finish the race?

Ⓐ 20 min   Ⓑ 33 min   Ⓒ 60 min   Ⓓ 120 min

Maintain Your Skills

Mixed Review In the figure, $\overline{QR} \parallel \overline{TS}$, $\overleftrightarrow{QT} \parallel \overleftrightarrow{RS}$, and $m\angle 1 = 131$.
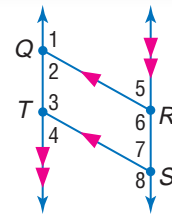**Find the measure of each angle.**   *(Lesson 3-2)*

51. $\angle 6$ **52.** $\angle 7$

53. $\angle 4$ **54.** $\angle 2$

55. $\angle 5$ **56.** $\angle 8$

State the transversal that forms each pair of angles. Then identify the special name for each angle pair.   *(Lesson 3-1)*

57. $\angle 1$ and $\angle 14$ **58.** $\angle 2$ and $\angle 10$

59. $\angle 3$ and $\angle 6$ **60.** $\angle 14$ and $\angle 15$

61. $\angle 7$ and $\angle 12$ **62.** $\angle 9$ and $\angle 11$
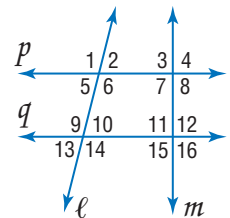
Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.   *(Lesson 2-1)*

63. Points H, I, and J are each located on different sides of a triangle.

64. Collinear points X, Y, and Z; Z is between X and Y.

65. $R(3, -4)$, $S(-2, -4)$, and $T(0, -4)$

Classify each angle as *right*, *acute*, or *obtuse*.   *(Lesson 1-4)*

66. $\angle ABD$ **67.** $\angle DBF$

68. $\angle CBE$ **69.** $\angle ABF$
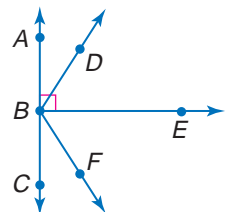
PREREQUISITE SKILL   Solve each equation for y.
*(To review **solving equations**, see pages 800 and 801.)*

70. $2x + y = 7$ **71.** $2x + 4y = -5$ **72.** $5x - 2y + 4 = 0$

CONTENTS

3-4 Equations of Lines

- Write an equation of a line given information about its graph.
- Solve problems by writing equations.

Vocabulary
- slope-intercept form
- point-slope form

How can the equation of a line describe the cost of cellular telephone service?

Cost of Cellular Service

A certain cellular phone company charges a flat rate of $19.95 per month for service. All calls are charged $0.07 per minute of air time t. The total charge C for a month can be represented by the equation $C = 0.07t + 19.95$.
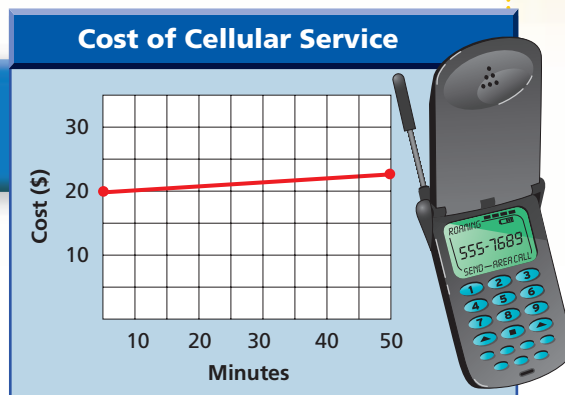
WRITE EQUATIONS OF LINES
You may remember from algebra that an equation of a line can be written given any of the following:

- the slope and the y-intercept,
- the slope and the coordinates of a point on the line, or
- the coordinates of two points on the line.

The graph of $C = 0.07t + 19.95$ has a slope of 0.07, and it intersects the y-axis at 19.95. These two values can be used to write an equation of the line. The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept.

$$y = mx + b \qquad C = 0.07t + 19.95$$

slope

y-intercept

Example 1 Slope and y-Intercept

Write an equation in slope-intercept form of the line with slope of -4 and y-intercept of 1.

$y = mx + b$ Slope-intercept form

$y = -4x + 1$ $m = -4, b = 1$

The slope-intercept form of the equation of the line is $y = -4x + 1$.

Another method used to write an equation of a line is the point-slope form of a linear equation. The **point-slope form** is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of any point on the line and m is the slope of the line.

given point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

slope

CONTENTS

**Choosing Forms
of Linear
Equations**
If you are given a point
on a line and the slope
of the line, use point-slope
form. Otherwise, use
slope-intercept form.

Example 2 *Slope and a Point*

Write an equation in point-slope form of the line whose slope is $-\frac{1}{2}$ that contains (3, −7).

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-7) = -\frac{1}{2}(x - 3)$ $m = -\frac{1}{2}, (x_1, y_1) = (3, -7)$

$y + 7 = -\frac{1}{2}(x - 3)$ Simplify.

The point-slope form of the equation of the line is $y + 7 = -\frac{1}{2}(x - 3)$.

Both the slope-intercept form and the point-slope form require the slope of a line in order to write an equation. There are occasions when the slope of a line is not given. In cases such as these, use two points on the line to calculate the slope. Then use the point-slope form to write an equation.

Writing Equations
Note that the point-slope
form of an equation is
different for each point
used. However, the slope-
intercept form of an
equation is unique.
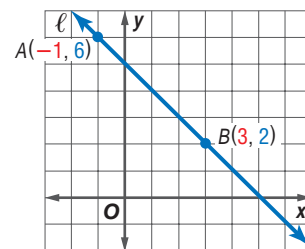
Example 3 *Two Points*

Write an equation in slope-intercept form for line ℓ.

Find the slope of ℓ by using $A(-1, 6)$ and $B(3, 2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \dfrac{2 - 6}{3 - (-1)}$ $x_1 = -1, x_2 = 3, y_1 = 6, y_2 = 2$

$= -\dfrac{4}{4}$ or -1 Simplify.

Now use the point-slope form and either point to write an equation.

Using Point A:

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 6 = -1[x - (-1)]$ $m = -1, (x_1, y_1) = (-1, 6)$

$y - 6 = -1(x + 1)$ Simplify.

$y - 6 = -x - 1$ Distributive Property

$y = -x + 5$ Add 6 to each side.

Using Point B:

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 2 = -1(x - 3)$ $m = -1, (x_1, y_1) = (3, 2)$

$y - 2 = -x + 3$ Distributive Property

$y = -x + 5$ Add 2 to each side.

Example 4 *One Point and an Equation*

Write an equation in slope-intercept form for a line containing (2, 0) that is perpendicular to the line $y = -x + 5$.

Since the slope of the line $y = -x + 5$ is −1, the slope of a line perpendicular to it is 1.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 0 = 1(x - 2)$ $m = 1, (x_1, y_1) = (2, 0)$

$y = x - 2$ Distributive Property

WRITE EQUATIONS TO SOLVE PROBLEMS Many real-world situations can be modeled using linear equations. In many business applications, the slope represents a rate.

Example 5 Write Linear Equations

CELL PHONE COSTS Martina's current cellular phone plan charges $14.95 per month and $0.10 per minute of air time.

a. Write an equation to represent the total monthly cost C for t minutes of air time.

For each minute of air time, the cost increases $0.10. So, the rate of change, or slope, is 0.10. The y-intercept is located where 0 minutes of air time are used, or $14.95.

$C = mt + b$ Slope-intercept form

$C = 0.10t + 14.95$ $m = 0.10, b = 14.95$

The total monthly cost can be represented by the equation $C = 0.10t + 14.95$.

b. Compare her current plan to the plan presented at the beginning of the lesson. If she uses an average of 40 minutes of air time each month, which plan offers the better rate?

Evaluate each equation for $t = 40$.

Current plan: $C = 0.10t + 14.95$

                 $= 0.10(40) + 14.95$ $t = 40$

                 $= 18.95$ Simplify.

Alternate plan: $C = 0.07t + 19.95$

                 $= 0.07(40) + 19.95$ $t = 40$

                 $= 22.75$ Simplify.

Given her average usage, Martina's current plan offers the better rate.

Check for Understanding

Concept Check

1. **Explain** how you would write an equation of a line whose slope is $-\frac{2}{5}$ that contains $(-2, 8)$.

2. **Write** equations in slope-intercept form for two lines that contain $(-1, -5)$.

3. **OPEN ENDED** Graph a line that is not horizontal or vertical on the coordinate plane. Write the equation of the line.

Guided Practice

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

4. $m = \frac{1}{2}$
y-intercept: 4

5. $m = -\frac{3}{5}$
intercept at $(0, -2)$

6. $m = 3$
y-intercept: -4

Write an equation in point-slope form of the line having the given slope that contains the given point.
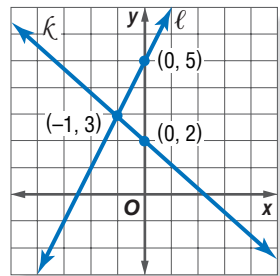
7. $m = \frac{3}{2}$, $(4, -1)$

8. $m = 3$, $(7, 5)$

9. $m = 1.25$, $(20, 137.5)$

CONTENTS

Refer to the figure at the right. Write an equation in slope-intercept form for each line.

10. ℓ

11. k

12. the line parallel to ℓ that contains (4, 4)

Application   **INTERNET   For Exercises 13–14, use the following information.**

Justin's current Internet service provider charges a flat rate of $39.95 per month for unlimited access. Another provider charges $4.95 per month for access and $0.95 for each hour of connection.

13. Write an equation to represent the total monthly cost for each plan.

14. If Justin is online an average of 60 hours per month, should he keep his current plan, or change to the other plan? Explain.

Practice and Apply

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

15. $m: \dfrac{1}{6}$, y-intercept: -4 **16.** $m: \dfrac{2}{3}$, (0, 8)

17. $m: \dfrac{5}{8}$, (0, -6)

18. $m: \dfrac{2}{9}$, y-intercept: $\dfrac{1}{3}$ **19.** $m: -1$, $b: -3$

20. $m: -\dfrac{1}{12}$, $b: 1$

Write an equation in point-slope form of the line having the given slope that contains the given point.

21. $m = 2$, (3, 1)   **22.** $m = -5$, (4, 7)   **23.** $m = -\dfrac{4}{5}$, (-12, -5)

24. $m = \dfrac{1}{16}$, (3, 11)   **25.** $m = 0.48$, (5, 17.12)   **26.** $m = -1.3$, (10, 87.5)

Write an equation in slope-intercept form for each line.

27. k

28. ℓ

29. m

30. n

31. perpendicular to line ℓ, contains (-1, 6)

32. parallel to line k, contains (7, 0)

33. parallel to line n, contains (0, 0)

34. perpendicular to line m, contains (-3, -3)
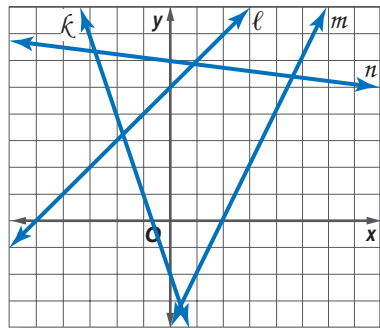
Write an equation in slope-intercept form for the line that satisfies the given conditions.

35. $m = -3$, y-intercept = 5   **36.** $m = 0$, y-intercept = 6

37. x-intercept = 5, y-intercept = 3   **38.** contains (4, -1) and (-2, -1)

39. contains (-5, -3) and (10, -6)   **40.** x-intercept = 5, y-intercept = -1

41. contains (-6, 8) and (-6, -4)   **42.** contains (-4, -1) and (-8, -5)

43. Write an equation of the line that contains (7, -2) and is parallel to $2x - 5y = 8$.

44. What is an equation of the line that is perpendicular to $2y + 2 = -\dfrac{7}{4}(x - 7)$ and contains (-2, -3)?
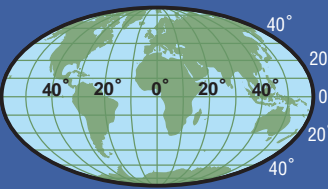
CONTENTS

45. JOBS Ann MacDonald is a salesperson at a discount appliance store. She earns $50 for each appliance that she sells plus a 5% commission on the price of the appliance. Write an equation that represents what she earned in a week in which she sold 15 appliances.

BUSINESS For Exercises 46–49, use the following information.
The Rainbow Paint Company sells an average of 750 gallons of paint each day.

46. How many gallons of paint will they sell in x days?

47. The store has 10,800 gallons of paint in stock. Write an equation in slope-intercept form that describes how many gallons of paint will be on hand after x days if no new stock is added.

48. Draw a graph that represents the number of gallons of paint on hand at any given time.

49. If it takes 4 days to receive a shipment of paint from the manufacturer after it is ordered, when should the store manager order more paint so that the store does not run out?
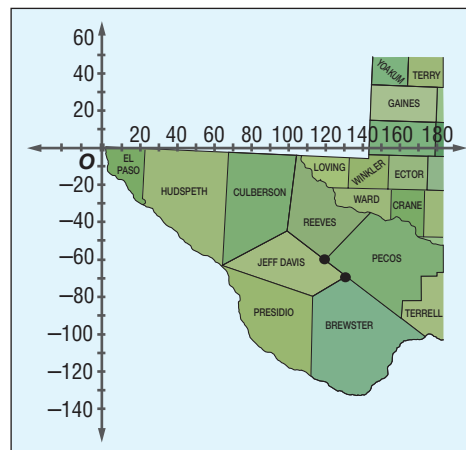
MAPS For Exercises 50 and 51, use the following information.
Many states have county maps that are grid-like in nature. If a map of Texas is placed on a coordinate plane with the western tip at the origin, the county lines can be represented by equations. Jeff Davis, Pecos, and Brewster Counties meet at (130, −70), and Jeff Davis, Reeves, and Pecos Counties meet at (120, −60).

50. Write an equation in slope-intercept form that models the county line between Jeff Davis and Reeves Counties.

51. The line separating Reeves and Pecos Counties runs perpendicular to the Jeff Davis/Reeves County line. Write an equation in slope-intercept form of the line that contains the Reeves/Pecos County line.

52. CRITICAL THINKING The point-slope form of an equation of a line can be rewritten as $y = m(x - x_1) + y_1$. Describe how the graph of $y = m(x - x_1) + y_1$ is related to the graph of $y = mx$.

53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can the equation of a line describe cellular telephone service?

Include the following in your answer:
- an explanation of how the fee for air time affects the equation, and
- a description of how you can use equations to compare various plans.

54. What is the slope of a line perpendicular to the line represented by $2x - 8y = 16$?
   Ⓐ -4 Ⓑ -2 Ⓒ $-\frac{1}{4}$ Ⓓ $\frac{1}{4}$

55. ALGEBRA What are all of the values of y for which $y^2 < 1$?
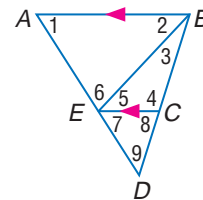   Ⓐ $y < -1$ Ⓑ $-1 < y < 1$ Ⓒ $y > -1$ Ⓓ $y < 1$

Mixed Review Determine the slope of the line that contains the given points. *(Lesson 3-3)*

56. $A(0, 6)$, $B(4, 0)$　　**57.** $G(8, 1)$, $H(8, -6)$　　**58.** $E(6, 3)$, $F(-6, 3)$

In the figure, $m\angle 1 = 58$, $m\angle 2 = 47$, and $m\angle 3 = 26$. Find the measure of each angle. *(Lesson 3-2)*
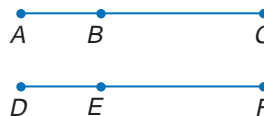
59. $\angle 7$　　　　**60.** $\angle 5$　　　　**61.** $\angle 6$

62. $\angle 4$　　　　**63.** $\angle 8$　　　　**64.** $\angle 9$

65. **PROOF**　Write a two-column proof. *(Lesson 2-6)*

Given: $AC = DF$
　　　　$AB = DE$

Prove: $BC = EF$

Find the perimeter of $\triangle ABC$ to the nearest hundredth, given the coordinates of its vertices. *(Lesson 1-6)*

66. $A(10, -6)$, $B(-2, -8)$, $C(-5, -7)$　　**67.** $A(-3, 2)$, $B(2, -9)$, $C(0, -10)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL　In the figure at the right, lines s and t are intersected by the transversal m. Name the pairs of angles that meet each description.
(To review angles formed by two lines and a transversal, see Lesson 3-1.)

68. consecutive interior angles

69. corresponding angles

70. alternate exterior angles

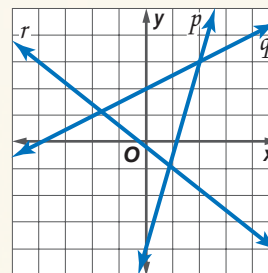**71.** alternate interior angles

---
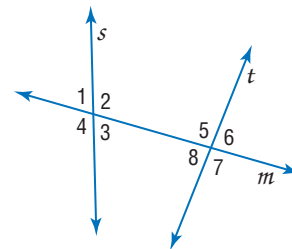
Practice Quiz 2　　　　　　　　Lessons 3-3 and 3-4

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. *(Lesson 3-3)*

1. $A(3, -1)$, $B(6, 1)$, $C(-2, -2)$, $D(2, 4)$　　　　**2.** $A(-3, -11)$, $B(3, 13)$, $C(0, -6)$, $D(8, -8)$

For Exercises 3–8, refer to the graph at the right. Find the slope of each line. *(Lesson 3-3)*

3. p

4. a line parallel to q

5. a line perpendicular to r

Write an equation in slope-intercept form for each line. *(Lesson 3-4)*

6. q

7. parallel to r, contains $(-1, 4)$

8. perpendicular to p, contains $(0, 0)$

Write an equation in point-slope form for the line that satisfies the given condition. *(Lesson 3-4)*

9. parallel to $y = -\frac{1}{4}x + 2$, contains $(5, -8)$

10. perpendicular to $y = -3$, contains $(-4, -4)$

CONTENTS

3-5 Proving Lines Parallel

What You'll Learn

- Recognize angle conditions that occur with parallel lines.
- Prove that two lines are parallel based on given angle relationships.

How do you know that the sides of a parking space are parallel?

Have you ever been in a tall building and looked down at a parking lot? The parking lot is full of line segments that appear to be parallel. The workers who paint these lines must be certain that they are parallel.

IDENTIFY PARALLEL LINES When each stripe of a parking space intersects the center line, the angles formed are corresponding angles. If the lines are parallel, we know that the corresponding angles are congruent. Conversely, if the corresponding angles are congruent, then the lines must be parallel.

Postulate 3.4

If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Abbreviation: *If corr. ∠s are ≅, then lines are ∥.*

Examples: If $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, or $\angle 4 \cong \angle 8$, then $m \parallel n$.
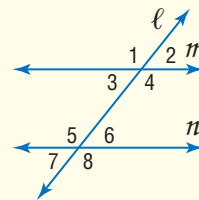

Postulate 3.4 justifies the construction of parallel lines.

Construction

Parallel Line Through a Point Not on Line

1 Use a straightedge to draw a line. Label two points on the line as M and N. Draw a point P that is not on $\overleftrightarrow{MN}$. Draw $\overleftrightarrow{PM}$.

2 Copy $\angle PMN$ so that P is the vertex of the new angle. Label the intersection points Q and R.

3 Draw $\overleftrightarrow{PQ}$. Because $\angle RPQ \cong \angle PMN$ by construction and they are corresponding angles, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{MN}$.

CONTENTS
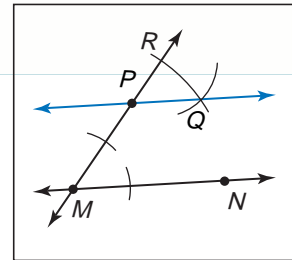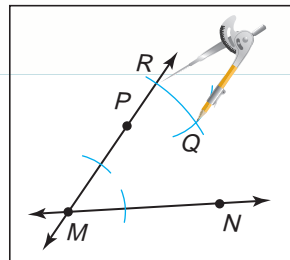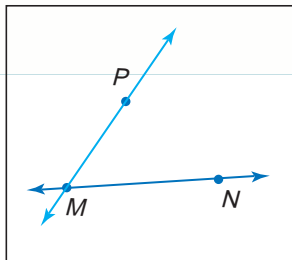
The construction establishes that there is *at least* one line through *P* that is parallel to $\overleftrightarrow{MN}$. In 1795, Scottish physicist and mathematician John Playfair provided the modern version of Euclid's Parallel Postulate, which states there is *exactly* one line parallel to a line through a given point not on the line.

Postulate 3.5

Parallel Postulate   If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

Parallel lines with a transversal create many pairs of congruent angles. Conversely, those pairs of congruent angles can determine whether a pair of lines is parallel.

Key Concept   *Proving Lines Parallel*

Theorems	Examples	
3.5 If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel. **Abbreviation:** *If alt. ext. ∠s are ≅, then lines are ∥.*	If ∠1 ≅ ∠8 or if ∠2 ≅ ∠7, then $m \parallel n$.	
3.6 If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel. **Abbreviation:** *If cons. int. ∠s are suppl., then lines are ∥.*	If $m\angle 3 + m\angle 5 = 180$ or if $m\angle 4 + m\angle 6 = 180$, then $m \parallel n$.	
3.7 If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel. **Abbreviation:** *If alt. int. ∠s are ≅, then lines are ∥.*	If ∠3 ≅ ∠6 or if ∠4 ≅ ∠5, then $m \parallel n$.	
3.8 In a plane, if two lines are perpendicular to the same line, then they are parallel. **Abbreviation:** *If 2 lines are ⊥ to the same line, then lines are ∥.*	If $\ell \perp m$ and $\ell \perp n$, then $m \parallel n$.	

Example 1 *Identify Parallel Lines*

In the figure, $\overline{BG}$ bisects $\angle ABH$. Determine which lines, if any, are parallel.

- The sum of the angle measures in a triangle must be 180, so $m\angle BDF = 180 - (45 + 65)$ or 70.

- Since $\angle BDF$ and $\angle BGH$ have the same measure, they are congruent.

- Congruent corresponding angles indicate parallel lines. So, $\overleftrightarrow{DF} \parallel \overleftrightarrow{GH}$.

- $\angle ABD \cong \angle DBF$, because $\overline{BG}$ bisects $\angle ABH$. So, $m\angle ABD = 45$.

- $\angle ABD$ and $\angle BDF$ are alternate interior angles, but they have different measures so they are not congruent.

- Thus, $\overleftrightarrow{AB}$ is not parallel to $\overleftrightarrow{DF}$ or $\overleftrightarrow{GH}$.
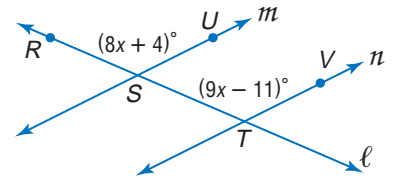
CONTENTS
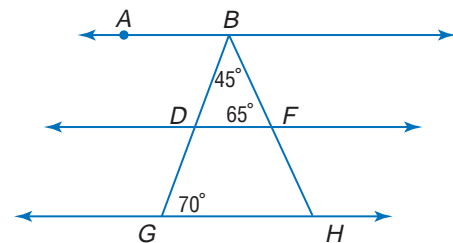
Angle relationships can be used to solve problems involving unknown values.

Example 2 Solve Problems with Parallel Lines

ALGEBRA Find x and $m\angle RSU$ so that $m \parallel n$.

Explore From the figure, you know that $m\angle RSU = 8x + 4$ and $m\angle STV = 9x - 11$. You also know that $\angle RSU$ and $\angle STV$ are corresponding angles.

Plan For line m to be parallel to line n, the corresponding angles must be congruent. So, $m\angle RSU = m\angle STV$. Substitute the given angle measures into this equation and solve for x. Once you know the value of x, use substitution to find $m\angle RSU$.

Solve

$m\angle RSU = m\angle STV$	Corresponding angles
$8x + 4 = 9x - 11$	Substitution
$4 = x - 11$	Subtract 8x from each side.
$15 = x$	Add 11 to each side.

Now use the value of x to find $m\angle RSU$.

$m\angle RSU = 8x + 4$	Original equation
$= 8(15) + 4$	$x = 15$
$= 124$	Simplify.

Examine Verify the angle measure by using the value of x to find $m\angle STV$. That is, $9x - 11 = 9(15) - 11$ or 124. Since $m\angle RSU = m\angle STV$, $\angle RSU \cong \angle STV$ and $m \parallel n$.
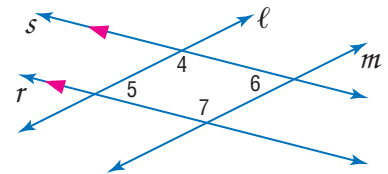

PROVE LINES PARALLEL
The angle pair relationships formed by a transversal can be used to prove that two lines are parallel.

Example 3 Prove Lines Parallel

Given: $r \parallel s$
$\angle 5 \cong \angle 6$

Prove: $\ell \parallel m$

Proof:

Statements	Reasons
1. $r \parallel s$, $\angle 5 \cong \angle 6$	1. Given
2. $\angle 4$ and $\angle 5$ are supplementary.	2. Consecutive Interior Angle Theorem
3. $m\angle 4 + m\angle 5 = 180$	3. Definition of supplementary angles
4. $m\angle 5 = m\angle 6$	4. Definition of congruent angles
5. $m\angle 4 + m\angle 6 = 180$	5. Substitution Property (=)
6. $\angle 4$ and $\angle 6$ are supplementary.	6. Definition of supplementary angles
7. $\ell \parallel m$	7. If cons. int. $\angle$s are suppl., then lines are $\parallel$.

In Lesson 3-3, you learned that parallel lines have the same slope. You can use the slopes of lines to prove that lines are parallel.
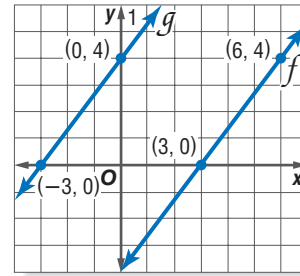
Example 4 Slope and Parallel Lines

Determine whether $g \parallel f$.

slope of f: $m = \dfrac{4 - 0}{6 - 3}$ or $\dfrac{4}{3}$

slope of g: $m = \dfrac{4 - 0}{0 - (-3)}$ or $\dfrac{4}{3}$

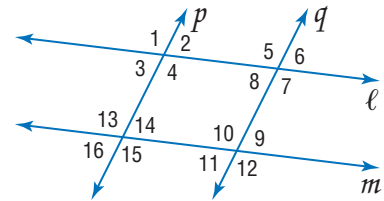Since the slopes are the same, $g \parallel f$.

Check for Understanding

Concept Check

1. **Summarize** five different methods to prove that two lines are parallel.

2. **Find a counterexample** for the following statement.
   If lines ℓ and m are cut by transversal t so that consecutive interior angles are congruent, then lines ℓ and m are parallel and t is perpendicular to both lines.

3. **OPEN ENDED** Describe two situations in your own life in which you encounter parallel lines. How could you verify that the lines are parallel?
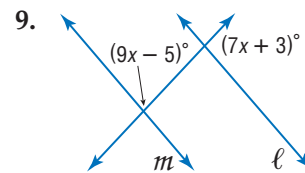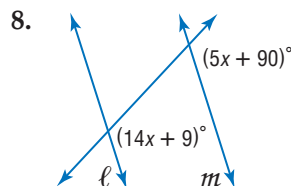
Guided Practice

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.
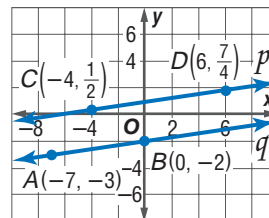
4. $\angle 16 \cong \angle 3$
5. $\angle 4 \cong \angle 13$
6. $m\angle 14 + m\angle 10 = 180$
7. $\angle 1 \cong \angle 7$

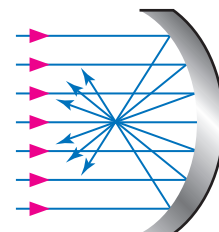Find x so that $\ell \parallel m$.

8.

9.

10. **PROOF** Write a two-column proof of Theorem 3.5.

11. Determine whether $p \parallel q$.

Application

12. **PHYSICS** The Hubble Telescope gathers parallel light rays and directs them to a central focal point. Use a protractor to measure several of the angles shown in the diagram. Are the lines parallel? Explain how you know.
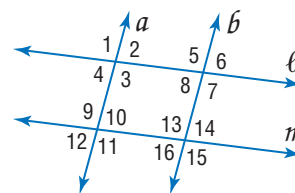
CONTENTS
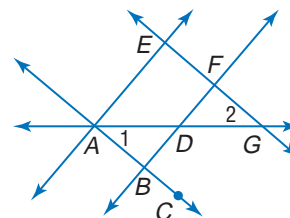
Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.
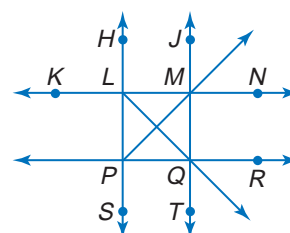
13. $\angle 2 \cong \angle 8$

14. $\angle 9 \cong \angle 16$

15. $\angle 2 \cong \angle 10$

16. $\angle 6 \cong \angle 15$

17. $\angle AEF \cong \angle BFG$

18. $\angle EAB \cong \angle DBC$

19. $\angle EFB \cong \angle CBF$

20. $m\angle GFD + m\angle CBD = 180$

21. $\angle HLK \cong \angle JML$

22. $\angle PLQ \cong \angle MQL$

23. $m\angle MLP + \angle RPL = 180$

24. $\overleftrightarrow{HS} \perp \overleftrightarrow{PR}, \overleftrightarrow{JT} \perp \overleftrightarrow{PR}$

25. **PROOF** Copy and complete the proof of Theorem 3.8.

Given: $\ell \perp t$
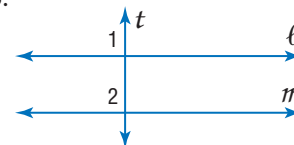$m \perp t$

Prove: $\ell \parallel m$

Proof:

Statements	Reasons
1. $\ell \perp t$, $m \perp t$	1. __?__
2. $\angle 1$ and $\angle 2$ are right angles.	2. __?__
3. $\angle 1 \cong \angle 2$	3. __?__
4. $\ell \parallel m$	4. __?__

Find x so that $\ell \parallel m$.

26. $(9x - 4)^\circ$, 140°

27. $(8x + 4)^\circ$, $(9x - 11)^\circ$

28. $(7x - 1)^\circ$

29. $(4 - 5x)^\circ$, $(7x + 100)^\circ$

30. $(14x + 9)^\circ$, $(5x + 90)^\circ$

31. $(178 - 3x)^\circ$, $(7x - 38)^\circ$
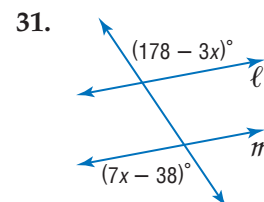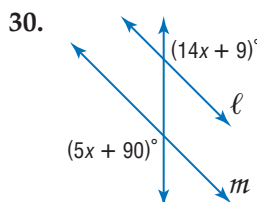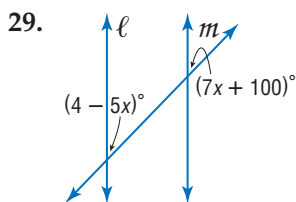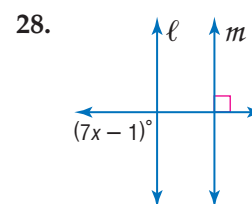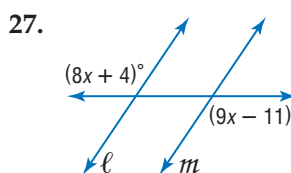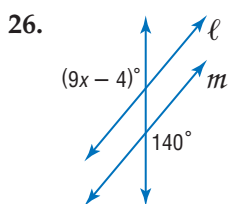
32. **PROOF** Write a two-column proof of Theorem 3.6.

33. **PROOF** Write a paragraph proof of Theorem 3.7.

PROOF   Write a two-column proof for each of the following.

34. Given: $\angle 2 \cong \angle 1$
$\qquad\qquad \angle 1 \cong \angle 3$
Prove: $\overline{ST} \parallel \overline{UV}$

35. Given: $\overline{AD} \perp \overline{CD}$
$\qquad\qquad \angle 1 \cong \angle 2$
Prove: $\overline{BC} \perp \overline{CD}$

36. Given: $\overline{JM} \parallel \overline{KN}$
$\qquad\qquad \angle 1 \cong \angle 2$
$\qquad\qquad \angle 3 \cong \angle 4$
Prove: $\overline{KM} \parallel \overline{LN}$

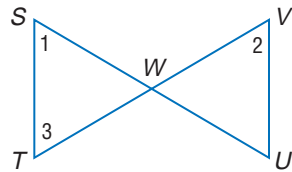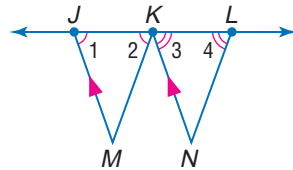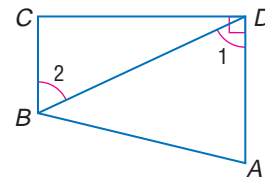**37. Given:** $\angle RSP \cong \angle PQR$
$\qquad\qquad \angle QRS$ and $\angle PQR$ are
$\qquad\qquad$ supplementary.
Prove: $\overline{PS} \parallel \overline{QR}$
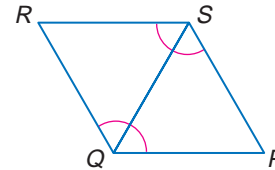
Determine whether each pair of lines is parallel. Explain why or why not.

38.

39.
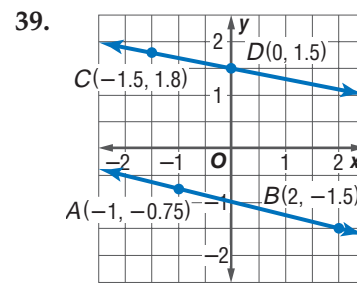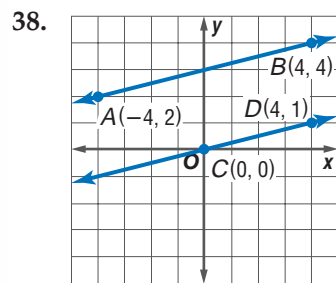
40. HOME IMPROVEMENT   To build a fence, Jim positioned the fence posts and then placed a 2×4 board at an angle between the fence posts. As he placed each picket, he measured the angle that the picket made with the 2×4. Why does this ensure that the pickets will be parallel?
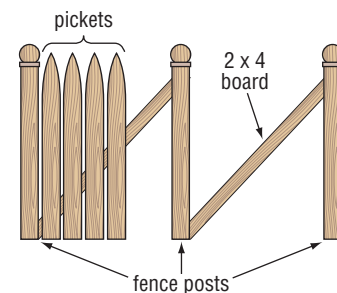
41. FOOTBALL   When striping the practice football field, Mr. Hawkinson first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel?

42. CRITICAL THINKING   When Adeel was working on an art project, he drew a four-sided figure with two pairs of opposite parallel sides. He noticed some patterns relating to the angles in the figure. List as many patterns as you can about a 4-sided figure with two pairs of opposite parallel sides.

43. RESEARCH   Use the Internet or other resource to find mathematicians like John Playfair who discovered new concepts and proved new theorems related to parallel lines. Briefly describe their discoveries.

CONTENTS

44. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How do you know that the sides of a parking space are parallel?

Include the following in your answer:
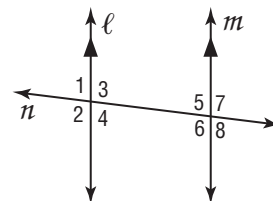- a comparison of the angles at which the lines forming the edges of a parking space strike the centerline, and
- a description of the type of parking spaces that form congruent consecutive interior angles.

45. In the figure, line ℓ is parallel to line m. Line n intersects both ℓ and m. Which of the following lists includes all of the angles that are supplementary to $\angle 1$?

Ⓐ angles 2, 3, and 4   Ⓑ angles 2, 3, 6, and 7

Ⓒ angles 4, 5, and 8   Ⓓ angles 3, 4, 7, and 8

46. **ALGEBRA**   Kendra has at least one quarter, one dime, one nickel, and one penny. If she has three times as many pennies as nickels, the same number of nickels as dimes, and twice as many dimes as quarters, then what is the least amount of money she could have?

Ⓐ $0.41   Ⓑ $0.48   Ⓒ $0.58   Ⓓ $0.61

Maintain Your Skills

Mixed Review   **Write an equation in slope-intercept form for the line that satisfies the given conditions.**   *(Lesson 3-4)*

47. $m = 0.3$, y-intercept is -6

48. $m = \frac{1}{3}$, contains $(-3, -15)$

49. contains $(5, 7)$ and $(-3, 11)$

50. perpendicular to $y = \frac{1}{2}x - 4$, contains $(4, 1)$

Find the slope of each line.   *(Lesson 3-3)*

51. $\overleftrightarrow{BD}$ **52.** $\overleftrightarrow{CD}$

53. $\overleftrightarrow{AB}$ **54.** $\overleftrightarrow{EO}$

55. any line parallel to $\overleftrightarrow{DE}$

56. any line perpendicular to $\overleftrightarrow{BD}$
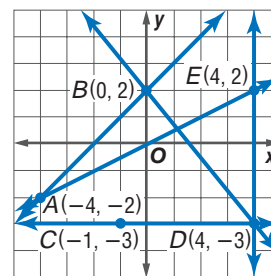
Construct a truth table for each compound statement.   *(Lesson 2-2)*

57. p and q **58.** p or $\sim q$ **59.** $\sim p \wedge q$ **60.** $\sim p \wedge \sim q$

61. **CARPENTRY**   A carpenter must cut two pieces of wood at angles so that they fit together to form the corner of a picture frame. What type of angles must he use to make sure that a corner results?   *(Lesson 1-5)*

Getting Ready for the Next Lesson   **PREREQUISITE SKILL   Use the Distance Formula to find the distance between each pair of points.**   *(To review **the Distance Formula**, see Lesson 1-4.)*

62. $(2, 7)$, $(7, 19)$ **63.** $(8, 0)$, $(-1, 2)$ **64.** $(-6, -4)$, $(-8, -2)$

www.geometryonline.com/self_check_quiz

CONTENTS

Graphing Calculator Investigation

Points of Intersection

You can use a TI-83 Plus graphing calculator to determine the points of intersection of a transversal and two parallel lines.

Example

Parallel lines ℓ and m are cut by a transversal t. The equations of ℓ, m, and t are $y = \frac{1}{2}x - 4$, $y = \frac{1}{2}x + 6$, and $y = -2x + 1$, respectively. Use a graphing calculator to determine the points of intersection of t with ℓ and m.
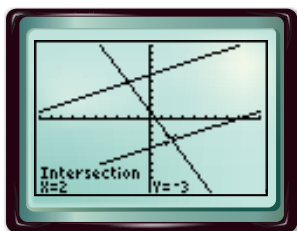
Step 1   Enter the equations in the Y= list and graph in the standard viewing window.

KEYSTROKES: Y= 1 ÷ 2 X,T,θ,n − 4 ENTER 1 ÷ 2 X,T,θ,n +

6 ENTER (−) 2 X,T,θ,n + 1 ZOOM 6

Step 2   Use the **CALC** menu to find the points of intersection.

- Find the intersection of ℓ and t.

  KEYSTROKES: 2nd [CALC] 5 ENTER ▼

  ENTER ENTER

  $[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

  Lines ℓ and t intersect at $(2, -3)$.

- Find the intersection of m and t.

  KEYSTROKES: 2nd [CALC] 5 ▼ ENTER

  ENTER ENTER
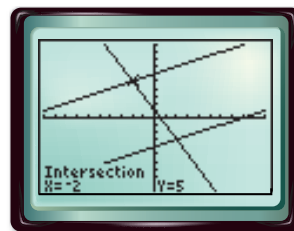
  $[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

  Lines m and t intersect at $(-2, 5)$.

Exercises

Parallel lines a and b are cut by a transversal t. Use a graphing calculator to determine the points of intersection of t with a and b. Round to the nearest tenth.

1. $a: y = 2x - 10$
   $b: y = 2x - 2$
   $t: y = -\frac{1}{2}x + 4$

2. $a: y = -x - 3$
   $b: y = -x + 5$
   $t: y = x - 6$

3. $a: y = 6$
   $b: y = 0$
   $t: x = -2$

4. $a: y = -3x + 1$
   $b: y = -3x - 3$
   $t: y = \frac{1}{3}x + 8$

5. $a: y = \frac{4}{5}x - 2$
   $b: y = \frac{4}{5}x - 7$
   $t: y = -\frac{5}{4}x$

6. $a: y = -\frac{1}{6}x + \frac{2}{3}$
   $b: y = -\frac{1}{6}x + \frac{5}{12}$
   $t: y = 6x + 2$

www.geometryonline.com/other_calculator_keystrokes

CONTENTS

3-6 Perpendiculars and Distance

What You'll Learn

- Find the distance between a point and a line.
- Find the distance between parallel lines.

Vocabulary
- equidistant

How does the distance between parallel lines relate to hanging new shelves?

When preparing to install new shelf brackets, it is important that the vertical bracing be parallel in order for the shelves to line up appropriately. One technique for ensuring that the braces are parallel is to install the first brace and then use a carpenter's square to measure and mark two or more points the same distance from the first brace. You can then align the second brace with those marks.

DISTANCE FROM A POINT TO A LINE In Lesson 3-5, you learned that if two lines are perpendicular to the same line, then they are parallel. The carpenter's square is used to construct a line perpendicular to each pair of shelves. The space between each pair of shelves is measured along the perpendicular segment. This is to ensure that the shelves are parallel. This relationship is an example of using lines and perpendicular segments to determine distance. The shortest segment from a point to a line is the perpendicular segment from the point to the line.

Key Concept — Distance Between a Point and a Line

- **Words** The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.
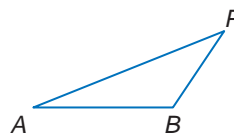
- **Model**
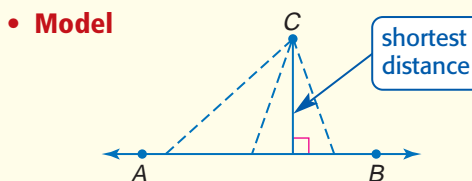
shortest distance

Study Tip

Measuring the Shortest Distance
You can use tools like the corner of a piece of paper or your book to help draw a right angle.

Example 1 Distance from a Point to a Line

Draw the segment that represents the distance from P to $\overleftrightarrow{AB}$.

Since the distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point, extend $\overline{AB}$ and draw $\overline{PQ}$ so that $\overline{PQ} \perp \overleftrightarrow{AB}$.
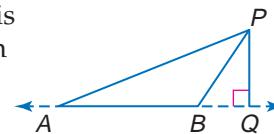
When you draw a perpendicular segment from a point to a line, you can guarantee that it is perpendicular by using the construction of a line perpendicular to a line through a point not on that line.

Aaron Haupt

CONTENTS

Example 2 — Construct a Perpendicular Segment

COORDINATE GEOMETRY Line ℓ contains points $(-6, -9)$ and $(0, -1)$.
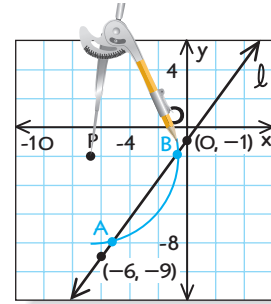Construct a line perpendicular to line ℓ through $P(-7, -2)$ not on ℓ.
Then find the distance from P to ℓ.

1 Graph line ℓ and point P. Place the compass
point at point P. Make the setting wide enough
so that when an arc is drawn, it intersects ℓ in two
places. Label these points of intersection A and B.

2 Put the compass at point A and draw an arc
below line ℓ. (*Hint*: Any compass setting
greater than $\frac{1}{2}AB$ will work.)
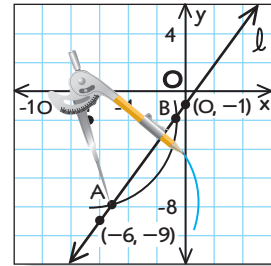
3 Using the same compass setting, put the
compass at point B and draw an arc to
intersect the one drawn in step 2. Label
the point of intersection Q.
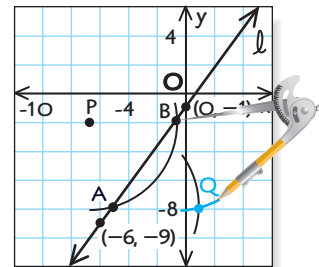
4 Draw $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ} \perp \ell$. Label point R at the intersection
of $\overleftrightarrow{PQ}$ and ℓ.   *Use the slopes of $\overleftrightarrow{PQ}$ and ℓ to verify
that the lines are perpendicular.*
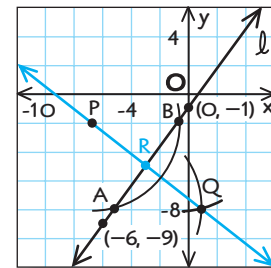
The segment constructed from point $P(-7, -2)$
perpendicular to the line ℓ, appears to intersect
line ℓ at $R(-3, -5)$. Use the Distance Formula
to find the distance between point P and line ℓ.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-7 - (-3))^2 + (-2 - (-5))^2}$$
$$= \sqrt{25} \text{ or } 5$$

The distance between P and ℓ is 5 units.

DISTANCE BETWEEN PARALLEL LINES

By definition, two parallel lines do not intersect. An
alternate definition states that two lines in a plane are
parallel if they are everywhere **equidistant**. Equidistant
means that the distance between two lines measured
along a perpendicular line to the lines is always the
same. To measure the distance between two parallel lines, measure the length of the
perpendicular segment whose endpoints lie on each of the two lines.

$AK = BJ = CH = DG = EF$

CONTENTS
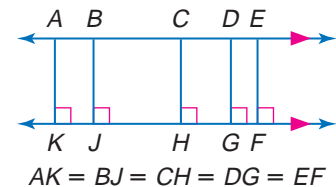
> **Key Concept**   **Distance Between Parallel Lines**
>
> The distance between two parallel lines is the distance between one of the lines and any point on the other line.

Recall that a *locus* is the set of all points that satisfy a given condition. Parallel lines can be described as the locus of points in a plane equidistant from a given line.

> **Theorem 3.9**
>
> In a plane, if two lines are equidistant from a third line, then the two lines are parallel to each other.
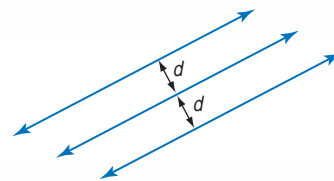
Example **3**   **Distance Between Lines**

Find the distance between the parallel lines ℓ and m whose equations are $y = -\frac{1}{3}x - 3$ and $y = -\frac{1}{3}x + \frac{1}{3}$, respectively.

You will need to solve a system of equations to find the endpoints of a segment that is perpendicular to both ℓ and m. The slope of lines ℓ and m is $-\frac{1}{3}$.

- First, write an equation of a line p perpendicular to ℓ and m. The slope of p is the opposite reciprocal of $-\frac{1}{3}$, or 3. Use the y-intercept of line ℓ, $(0, -3)$, as one of the endpoints of the perpendicular segment.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) &&\text{Point-slope form}\\
y - (-3) &= 3(x - 0) &&x_1 = 0, y_1 = -3, m = 3\\
y + 3 &= 3x &&\text{Simplify.}\\
y &= 3x - 3 &&\text{Subtract 3 from each side.}
\end{aligned}
$$
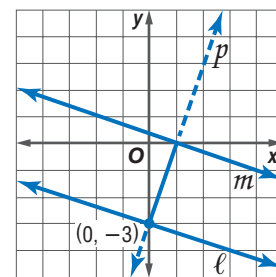
- Next, use a system of equations to determine the point of intersection of line m and p.

m: $y = -\frac{1}{3}x + \frac{1}{3}$

p: $y = 3x - 3$

$$
\begin{aligned}
-\frac{1}{3}x + \frac{1}{3} &= 3x - 3 &&\text{Substitute } -\frac{1}{3}x + \frac{1}{3} \text{ for } y \text{ in the second equation.}\\
-\frac{1}{3}x - 3x &= -3 - \frac{1}{3} &&\text{Group like terms on each side.}\\
-\frac{10}{3}x &= -\frac{10}{3} &&\text{Simplify on each side.}\\
x &= 1 &&\text{Divide each side by } -\frac{10}{3}.\\
y &= 3(1) - 3 &&\text{Substitute 1 for } x \text{ in the equation for } p.\\
y &= 0 &&\text{Simplify.}
\end{aligned}
$$

The point of intersection is $(1, 0)$.

- Then, use the Distance Formula to determine the distance between $(0, -3)$ and $(1, 0)$.

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} &&\text{Distance Formula}\\
&= \sqrt{(0 - 1)^2 + (-3 - 0)^2} &&x_2 = 0, x_1 = 1, y_2 = -3, y_1 = 0\\
&= \sqrt{10} &&\text{Simplify.}
\end{aligned}
$$

The distance between the lines is $\sqrt{10}$ or about 3.16 units.

Check for Understanding

Concept Check

1. **Explain** how to construct a segment between two parallel lines to represent the distance between them.

2. **OPEN ENDED** Make up a problem involving an everyday situation in which you need to find the distance between a point and a line or the distance between two lines. For example, find the shortest path from the patio of a house to a garden to minimize the length of a walkway and material used in its construction.

3. **Compare and contrast** three different methods that you can use to show that two lines in a plane are parallel.
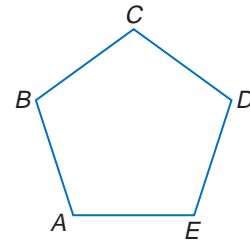
Guided Practice

Copy each figure. Draw the segment that represents the distance indicated.

4. L to $\overleftrightarrow{KN}$
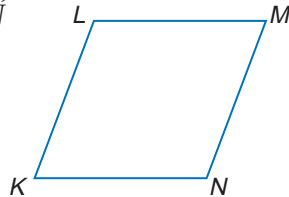
5. D to $\overleftrightarrow{AE}$

6. **COORDINATE GEOMETRY** Line ℓ contains points $(0, 0)$ and $(2, 4)$. Draw line ℓ. Construct a line perpendicular to ℓ through $A(2, -6)$. Then find the distance from A to ℓ.

Find the distance between each pair of parallel lines.

7. $y = \dfrac{3}{4}x - 1$

   $y = \dfrac{3}{4}x + \dfrac{1}{8}$

8. $x + 3y = 6$

   $x + 3y = -14$

9. Graph the line whose equation is $y = -\dfrac{3}{4}x + \dfrac{1}{4}$. Construct a perpendicular segment through $P(2, 5)$. Then find the distance from P to the line.

Application

10. **UTILITIES** Housing developers often locate the shortest distance from a house to the water main so that a minimum of pipe is required to connect the house to the water supply. Copy the diagram, and draw a possible location for the pipe.
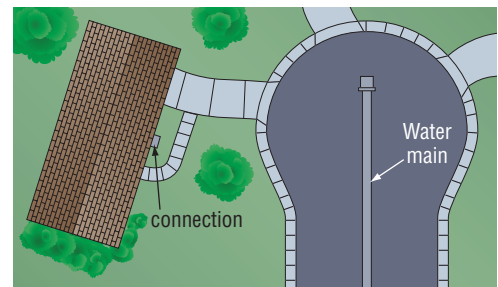
Water main

connection

Practice and Apply

Copy each figure. Draw the segment that represents the distance indicated.

11. C to $\overleftrightarrow{AD}$

12. K to $\overleftrightarrow{JL}$

13. Q to $\overleftrightarrow{RS}$

CONTENTS

Copy each figure. Draw the segment that represents the distance indicated.

14. Y to $\overleftrightarrow{WX}$
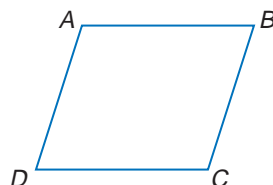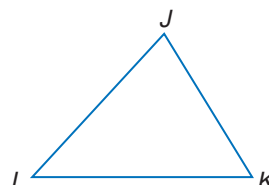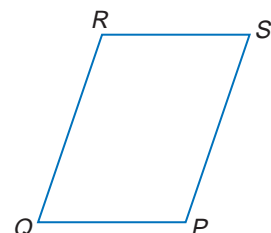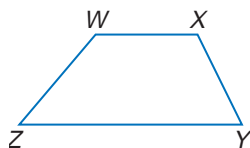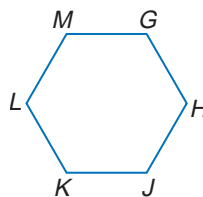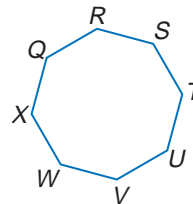
15. G to $\overleftrightarrow{HJ}$

16. W to $\overleftrightarrow{UV}$

COORDINATE GEOMETRY Construct a line perpendicular to ℓ through P. Then find the distance from P to ℓ.

17. Line ℓ contains points $(-3, 0)$ and $(3, 0)$. Point P has coordinates $(4, 3)$.

18. Line ℓ contains points $(0, -2)$ and $(1, 3)$. Point P has coordinates $(-4, 4)$.

Find the distance between each pair of parallel lines.

19. $y = -3$
$y = 1$

20. $x = 4$
$x = -2$

21. $y = 2x + 2$
$y = 2x - 3$

22. $y = 4x$
$y = 4x - 17$

23. $y = 2x - 3$
$2x - y = -4$

24. $y = -\dfrac{3}{4}x - 1$
$3x + 4y = 20$

Graph each line. Construct a perpendicular segment through the given point. Then find the distance from the point to the line.

25. $y = 5, (-2, 4)$

26. $y = 2x + 2, (-1, -5)$

27. $2x - 3y = -9, (2, 0)$

28. **PROOF** Write a paragraph proof of Theorem 3.9.

29. INTERIOR DESIGN Theresa is installing a curtain rod on the wall above the window. In order to ensure that the rod is parallel to the ceiling, she measures and marks 9 inches below the ceiling in several places. If she installs the rod at these markings centered over the window, how does she know the curtain rod will be parallel to the ceiling?

30. CONSTRUCTION When framing a wall during a construction project, carpenters often use a plumb line. A *plumb line* is a string with a weight called a *plumb bob* attached on one end. The plumb line is suspended from a point and then used to ensure that wall studs are vertical. How does the plumb line help to find the distance from a point to the floor?
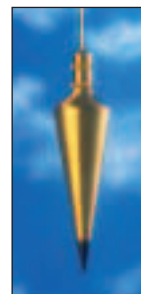
31. ALGEBRA In the coordinate plane, if a line has equation $ax + by = c$, then the distance from a point (x_1, y_1) is given by $\dfrac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$. Determine the distance from $(4, 6)$ to the line whose equation is $3x + 4y = 6$.

32. CRITICAL THINKING Draw a diagram that represents each description.
  a. Point P is equidistant from two parallel lines.
  b. Point P is equidistant from two intersecting lines.
  c. Point P is equidistant from two parallel planes.
  d. Point P is equidistant from two intersecting planes.
  e. A line is equidistant from two parallel planes.
  f. A plane is equidistant from two other planes that are parallel.

33. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How does the distance between parallel lines relate to hanging new shelves?

Include the following in your answer:

- an explanation of why marking several points equidistant from the first brace will ensure that the braces are parallel, and
- a description of other types of home improvement projects that require that two or more elements are parallel.

34. GRID IN Segment AB is perpendicular to segment BD. Segment AB and segment CD bisect each other at point X. If $AB = 16$ and $CD = 20$, what is the length of $\overline{BD}$?

35. ALGEBRA A coin was flipped 24 times and came up heads 14 times and tails 10 times. If the first and the last flips were both heads, what is the greatest number of consecutive heads that could have occurred?

Ⓐ 7   Ⓑ 9   Ⓒ 10   Ⓓ 13

Maintain Your Skills

Mixed Review **Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.** *(Lesson 3-5)*

36. $\angle 5 \cong \angle 6$

37. $\angle 6 \cong \angle 2$

38. $\angle 1$ and $\angle 2$ are supplementary.
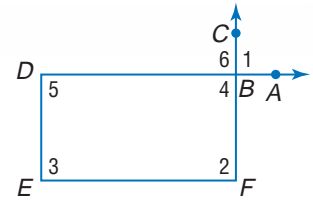
Write an equation in slope-intercept form for each line.
(Lesson 3-4)

39. a **40.** b **41.** c

42. perpendicular to line a, contains $(-1, -4)$

43. parallel to line c, contains $(2, 5)$
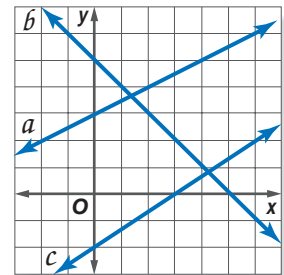
44. PROOF Write a two-column proof. *(Lesson 2-7)*

Given: $NL = NM$
          $AL = BM$

Prove: $NA = NB$
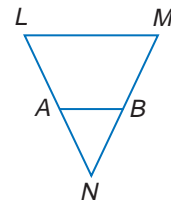
WebQuest **Internet Project** *When Is Weather Normal?*

It's time to complete your project. Use the information and data you have gathered about climate and locations on Earth to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

www.geometryonline.com/webquest

CONTENTS

Non-Euclidean Geometry

So far in this text, we have studied **plane Euclidean geometry**, which is based on a system of points, lines, and planes. In **spherical geometry**, we study a system of points, great circles (lines), and spheres (planes). Spherical geometry is one type of **non-Euclidean geometry**.

Plane Euclidean Geometry

Plane $\mathcal{P}$ contains line ℓ and point A not on ℓ.

Spherical Geometry

Longitude lines and the equator model great circles on Earth.

A great circle divides a sphere into equal halves.

Polar points are endpoints of a diameter of a great circle.

Sphere $\mathcal{E}$ contains great circle m and point P not on m. m is a line on sphere $\mathcal{E}$.

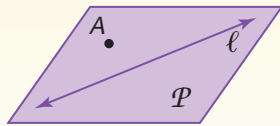The table below compares and contrasts *lines* in the system of plane Euclidean geometry and *lines* (great circles) in spherical geometry.

Plane Euclidean Geometry Lines on the Plane	Spherical Geometry Great Circles (Lines) on the Sphere
1. A line segment is the shortest path between two points.	1. An arc of a great circle is the shortest path between two points.
2. There is a unique line passing through any two points.	2. There is a unique great circle passing through any pair of nonpolar points.
3. A line goes on infinitely in two directions.	3. A great circle is finite and returns to its original starting point.
4. If three points are collinear, exactly one is between the other two. B is between A and C.	4. If three points are collinear, any one of the three points is between the other two. A is between B and C. B is between A and C. C is between A and B.

In spherical geometry, Euclid's first four postulates and their related theorems hold true. However, theorems that depend on the parallel postulate (Postulate 5) may not be true.

In Euclidean geometry parallel lines lie in the same plane and never intersect. In spherical geometry, the sphere is the plane, and a great circle represents a line. Every great circle containing A intersects ℓ. Thus, there exists no line through point A that is parallel to ℓ.
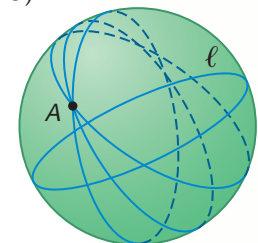
(continued on the next page)

CONTENTS

Geometry Activity

Every great circle of a sphere intersects all other great circles on that sphere in exactly two points. In the figure at the right, one possible line through point A intersects line ℓ at P and Q.
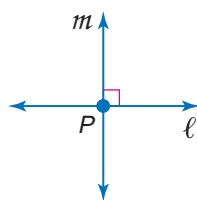
If two great circles divide a sphere into four congruent regions, the lines are perpendicular to each other at their intersection points. Each longitude circle on Earth intersects the equator at right angles.
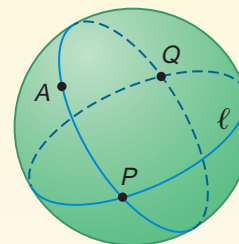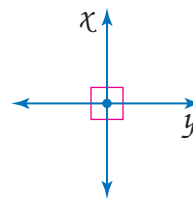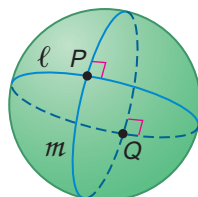
Example *Compare Plane and Spherical Geometries*

For each property listed from plane Euclidean geometry, write a corresponding statement for spherical geometry.

a. Perpendicular lines intersect at one point.

Perpendicular great circles intersect at two points.

b. Perpendicular lines form four right angles.

Perpendicular great circles form eight right angles.
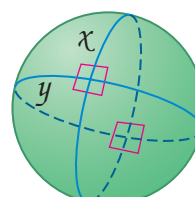
Exercises

For each property from plane Euclidean geometry, write a corresponding statement for spherical geometry.

1. A line goes on infinitely in two directions.
2. A line segment is the shortest path between two points.
3. Two distinct lines with no point of intersection are parallel.
4. Two distinct intersecting lines intersect in exactly one point.
5. A pair of perpendicular straight lines divides the plane into four infinite regions.
6. Parallel lines have infinitely many common perpendicular lines.
7. There is only one distance that can be measured between two points.

If spherical points are restricted to be nonpolar points, determine if each statement from plane Euclidean geometry is also *true* in spherical geometry. If *false*, explain your reasoning.

8. Any two distinct points determine exactly one line.
9. If three points are collinear, exactly one point is between the other two.
10. Given a line ℓ and point P not on ℓ, there exists exactly one line parallel to ℓ passing through P.

CONTENTS

Vocabulary and Concept Check

alternate exterior angles (p. 128)
alternate interior angles (p. 128)
consecutive interior angles (p. 128)
corresponding angles (p. 128)
equidistant (p. 160)
non-Euclidean geometry (p. 165)

parallel lines (p. 126)
parallel planes (p. 126)
plane Euclidean geometry (p. 165)
point-slope form (p. 145)
rate of change (p. 140)

skew lines (p. 127)
slope (p. 139)
slope-intercept form (p. 145)
spherical geometry (p. 165)
transversal (p. 127)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises Refer to the figure and choose the term that best completes each sentence.

1. Angles 4 and 5 are *(consecutive, alternate)* interior angles.
2. The distance from point A to line n is the length of the segment *(perpendicular, parallel)* to line n through A.
3. If $\angle 4$ and $\angle 6$ are supplementary, lines m and n are said to be *(parallel, intersecting)* lines.
4. Line ℓ is a *(slope-intercept, transversal)* for lines n and m.
5. $\angle 1$ and $\angle 8$ are *(alternate interior, alternate exterior)* angles.
6. If $n \parallel m$, $\angle 6$ and $\angle 3$ are *(supplementary, congruent)*.
7. Angles 5 and 3 are *(consecutive, alternate)* interior angles.
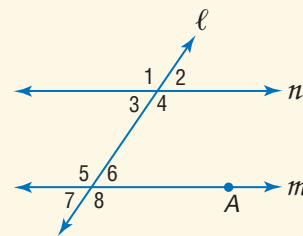
Lesson-by-Lesson Review

3-1 Parallel Lines and Transversals

See pages 126–131.

Concept Summary
- Coplanar lines that do not intersect are called *parallel*.
- When two lines are cut by a transversal, there are many angle relationships.

Example Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

a. $\angle 7$ and $\angle 3$
   corresponding
b. $\angle 4$ and $\angle 6$
   consecutive interior
c. $\angle 7$ and $\angle 2$
   alternate exterior
d. $\angle 3$ and $\angle 6$
   alternate interior
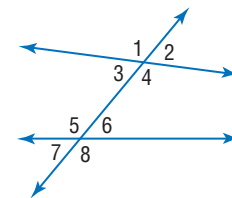
Exercises Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles. *See Example 3 on page 128.*

8. $\angle 10$ and $\angle 6$
9. $\angle 5$ and $\angle 12$
10. $\angle 8$ and $\angle 10$
11. $\angle 1$ and $\angle 9$
12. $\angle 3$ and $\angle 6$
13. $\angle 5$ and $\angle 3$
14. $\angle 2$ and $\angle 7$
15. $\angle 8$ and $\angle 9$

www.geometryonline.com/vocabulary_review
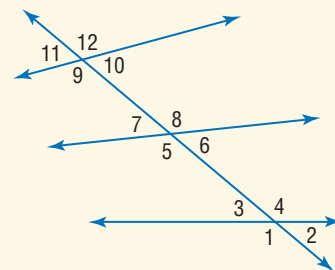
3-2   Angles and Parallel Lines

Concept Summary

- Pairs of congruent angles formed by parallel lines and a transversal are corresponding angles, alternate interior angles, and alternate exterior angles.
- Pairs of consecutive interior angles are supplementary.

Example

In the figure, $m\angle 1 = 4p + 15$, $m\angle 3 = 3p - 10$, and $m\angle 4 = 6r + 5$. Find the values of p and r.

- Find p.

  Since $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$, $\angle 1$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem.

$m\angle 1 + m\angle 3 = 180$	Definition of supplementary angles
$(4p + 15) + (3p - 10) = 180$	Substitution
$7p + 5 = 180$	Simplify.
$p = 25$	Solve for p.

- Find r.

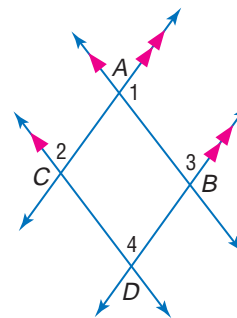 Since $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\angle 4 \cong \angle 3$ by the Corresponding Angles Postulate.

$m\angle 4 = m\angle 3$	Definition of congruent angles
$6r + 5 = 3(25) - 10$	Substitution, $p = 25$
$6r + 5 = 65$	Simplify.
$r = 10$	Solve for x.

Exercises   In the figure, $m\angle 1 = 53$. Find the measure of each angle.   *See Example 1 on page 133.*

16. $\angle 2$
17. $\angle 3$
18. $\angle 4$
19. $\angle 5$
20. $\angle 6$
21. $\angle 7$
22. In the figure, $m\angle 1 = 3a + 40$, $m\angle 2 = 2a + 25$, and $m\angle 3 = 5b - 26$. Find a and b.
See Example 3 on page 135.
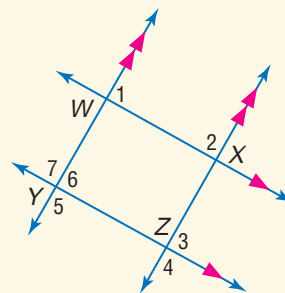
3-3   Slopes of Lines

Concept Summary

- The slope of a line is the ratio of its vertical rise to its horizontal run.
- Parallel lines have the same slope, while perpendicular lines have slopes whose product is -1.

Example

Determine whether $\overleftrightarrow{KM}$ and $\overleftrightarrow{LN}$ are *parallel*, *perpendicular*, or *neither* for $K(-3, 3)$, $M(-1, -3)$, $L(2, 5)$, and $N(5, -4)$.

slope of $\overleftrightarrow{KM}$: $m = \dfrac{-3 - 3}{-1 - (-3)}$ or -3 slope of $\overleftrightarrow{LN}$: $m = \dfrac{-4 - 5}{5 - 2}$ or -3

The slopes are the same. So $\overleftrightarrow{KM}$ and $\overleftrightarrow{LN}$ are parallel.

CONTENTS

Exercises   Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel, perpendicular,* or *neither.*
See Example 3 on page 141.

23. $A(-4, 1), B(3, -1), C(2, 2), D(0, 9)$
24. $A(6, 2), B(2, -2), C(-1, -4), D(5, 2)$
25. $A(1, -3), B(4, 5), C(1, -1), D(-7, 2)$
26. $A(2, 0), B(6, 3), C(-1, -4), D(3, -1)$

Graph the line that satisfies each condition.   *See Example 4 on page 141.*
27. contains $(2, 3)$ and is parallel to $\overleftrightarrow{AB}$ with $A(-1, 2)$ and $B(1, 6)$
28. contains $(-2, -2)$ and is perpendicular to $\overleftrightarrow{PQ}$ with $P(5, 2)$ and $Q(3, -4)$

3-4 Equations of Lines

Concept Summary

In general, an equation of a line can be written if you are given:

- slope and the y-intercept
- the slope and the coordinates of a point on the line, or
- the coordinates of two points on the line.

Example **Write an equation in slope-intercept form of the line that passes through $(2, -4)$ and $(-3, 1)$.**

Find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{1 - (-4)}{-3 - 2} \qquad \begin{array}{l}(x_1, y_1) = (2, -4),\\ (x_2, y_2) = (-3, 1)\end{array}$$

$$= \frac{5}{-5} \text{ or } -1 \quad \text{Simplify.}$$

Now use the point-slope form and either point to write an equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - (-4) = -1(x - 2) \quad m = -1, (x_1, y_1) = (2, -4)$$

$$y + 4 = -x + 2 \qquad \text{Simplify.}$$

$$y = -x - 2 \qquad \text{Subtract 4 from each side.}$$

Exercises   Write an equation in slope-intercept form of the line that satisfies the given conditions.   *See Examples 1–3 on pages 145 and 146.*

29. $m = 2$, contains $(1, -5)$
30. contains $(2, 5)$ and $(-2, -1)$
31. $m = -\frac{2}{7}$, y-intercept $= 4$
32. $m = -\frac{3}{2}$, contains $(2, -4)$
33. $m = 5$, y-intercept $= -3$
34. contains $(3, -1)$ and $(-4, 6)$

## 3-5 Proving Lines Parallel

Concept Summary

When lines are cut by a transversal, certain angle relationships produce parallel lines.

- congruent corresponding angles
- congruent alternate interior angles
- congruent alternate exterior angles
- supplementary consecutive interior angles

CONTENTS

Chapter

3 For More ...

- Extra Practice, see pages 768–770.
- Mixed Problem Solving, see page 781.

Example **If ∠1 ≅ ∠8, which lines if any are parallel?**

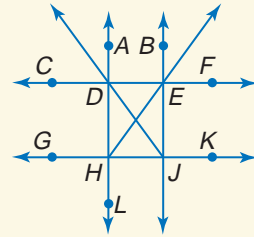∠1 and ∠8 are alternate exterior angles for lines r and s. These lines are cut by the transversal p. Since the angles are congruent, lines r and s are parallel by Theorem 3.5.
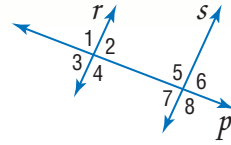
Exercises Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer. *See Example 1 on page 152.*

35. $\angle GHL \cong \angle EJK$

36. $m\angle ADJ + m\angle DJE = 180$

37. $\overleftrightarrow{CF} \perp \overleftrightarrow{AL}, \overleftrightarrow{GK} \perp \overleftrightarrow{AL}$

38. $\angle DJE \cong \angle HDJ$

39. $m\angle EJK + m\angle JEF = 180$

40. $\angle GHL \cong \angle CDH$

3-6 *Perpendiculars and Distance*

See pages 159–164.

Concept Summary

- The distance between a point and a line is measured by the perpendicular segment from the point to the line.

Example **Find the distance between the parallel lines q and r whose equations are $y = x - 2$ and $y = x + 2$, respectively.**

- The slope of q is 1. Choose a point on line q such as $P(2, 0)$. Let line k be perpendicular to q through P. The slope of line k is -1. Write an equation for line k.

$y = mx + b$ Slope-intercept form

$0 = (-1)(2) + b$ $y = 0, m = -1, x = 2$

$2 = b$ Solve for *b*.   An equation for k is $y = -x + 2$.

- Use a system of equations to determine the point of intersection of k and r.

$y = \ \ x + 2$

$\underline{y = -x + 2}$

$2y = \qquad 4$ Add the equations.

$y = 2$ Divide each side by 2.

Substitute 2 for y in the original equation.

$2 = -x + 2$

$x = 0$ Solve for *x*.

The point of intersection is $(0, 2)$.

- Now use the Distance Formula to determine the distance between $(2, 0)$ and $(0, 2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 0)^2 + (0 - 2)^2} = \sqrt{8}$$

The distance between the lines is $\sqrt{8}$ or about 2.83 units.

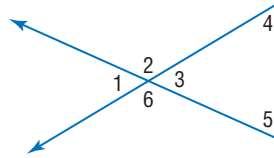**Exercises** Find the distance between each pair of parallel lines. *See Example 3 on page 161.*

41. $y = 2x - 4, y = 2x + 1$

42. $y = \frac{1}{2}x, y = \frac{1}{2}x + 5$

CONTENTS

Chapter 3 Practice Test
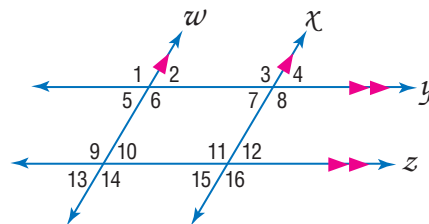
Vocabulary and Concepts

1. Write an equation of a line that is perpendicular to $y = 3x - \frac{2}{7}$.
2. Name a theorem that can be used to prove that two lines are parallel.
3. Find all the angles that are supplementary to $\angle 1$.

Skills and Applications

In the figure, $m\angle 12 = 64$. Find the measure of each angle.

4. $\angle 8$
5. $\angle 13$
6. $\angle 7$
7. $\angle 11$
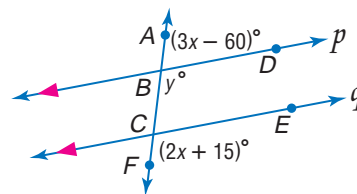8. $\angle 3$
9. $\angle 4$
10. $\angle 9$
11. $\angle 5$

Graph the line that satisfies each condition.

12. slope $= -1$, contains $P(-2, 1)$
13. contains $Q(-1, 3)$ and is perpendicular to $\overleftrightarrow{AB}$ with $A(-2, 0)$ and $B(4, 3)$
14. contains $M(1, -1)$ and is parallel to $\overleftrightarrow{FG}$ with $F(3, 5)$ and $G(-3, -1)$
15. slope $= -\frac{4}{3}$, contains $K(3, -2)$

For Exercises 16–21, refer to the figure at the right. Find each value if $p \parallel q$.

16. x
17. y
18. $m\angle FCE$
19. $m\angle ABD$
20. $m\angle BCE$
21. $m\angle CBD$

Find the distance between each pair of parallel lines.

22. $y = 2x - 1, y = 2x + 9$
23. $y = -x + 4, y = -x - 2$

24. **COORDINATE GEOMETRY** Detroit Road starts in the center of the city, and Lorain Road starts 4 miles west of the center of the city. Both roads run southeast. If these roads are put on a coordinate plane with the center of the city at $(0, 0)$, Lorain Road is represented by the equation $y = -x - 4$ and Detroit Road is represented by the equation $y = -x$. How far away is Lorain Road from Detroit Road?

25. **STANDARDIZED TEST PRACTICE** In the figure at the right, which cannot be true if $m \parallel \ell$ and $m\angle 1 = 73$?
    - (A) $m\angle 4 > 73$
    - (B) $\angle 1 \cong \angle 4$
    - (C) $m\angle 2 + m\angle 3 = 180$
    - (D) $\angle 3 \cong \angle 1$
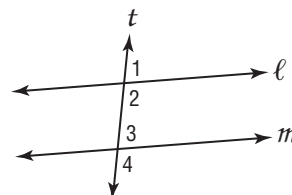
Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Jahaira needed 2 meters of fabric to reupholster a chair in her bedroom. If Jahaira can only find a centimeter ruler, how much fabric should she cut? (Prerequisite Skill)
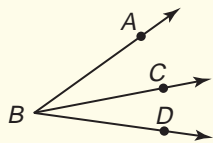
   (A) 20 cm   (B) 200 cm
   (C) 2000 cm   (D) 20,000 cm

2. A fisherman uses a coordinate grid marked in miles to locate the nets cast at sea. How far apart are nets *A* and *B*? (Lesson 1-3)

   (A) 3 mi
   (B) $\sqrt{28}$ mi
   (C) $\sqrt{65}$ mi
   (D) 11 mi
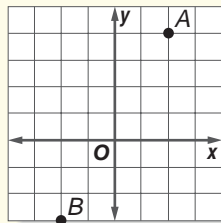
3. If $\angle ABC \cong \angle CBD$, which statement must be true? (Lesson 1-5)

   (A) Segment *BC* bisects $\angle ABD$.
   (B) $\angle ABD$ is a right angle.
   (C) $\angle ABC$ and $\angle CBD$ are supplementary.
   (D) Segments *AB* and *BD* are perpendicular.

4. Valerie cut a piece of wood at a 72° angle for her project. What is the degree measure of the supplementary angle on the leftover piece of wood? (Lesson 1-6)

   (A) 18   (B) 78   (C) 98   (D) 108

5. A pan balance scale is often used in science classes. What is the value of *x* to balance the scale if one side weighs $4x + 4$ units and the other weighs $6x - 8$ units? (Lesson 2-3)

   (A) 1   (B) 2   (C) 3   (D) 6

Use the diagram below of a tandem bicycle frame for Questions 6 and 7.
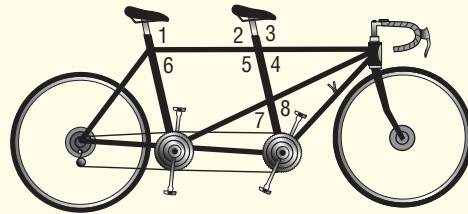
6. The diagram shows the two posts on which seats are placed and several crossbars. Which term describes $\angle 6$ and $\angle 5$? (Lesson 3-1)

   (A) alternate exterior angles
   (B) alternate interior angles
   (C) consecutive interior angles
   (D) corresponding angles

7. The quality control manager for the bicycle manufacturer wants to make sure that the two seat posts are parallel. Which angles can she measure to determine this? (Lesson 3-5)

   (A) $\angle 2$ and $\angle 3$ (B) $\angle 1$ and $\angle 3$
   (C) $\angle 4$ and $\angle 8$ (D) $\angle 5$ and $\angle 7$

8. Which is the equation of a line that is perpendicular to the line $4y - x = 8$? (Lesson 3-3)

   (A) $y = -\frac{1}{4}x - 2$ (B) $y = \frac{1}{4}x + 2$
   (C) $y = -4x - 15$ (D) $y = 4x + 15$

9. The graph of $y = 2x - 5$ is shown at the right. How would the graph be different if the number 2 in the equation was replaced with a 4? (Lesson 3-4)
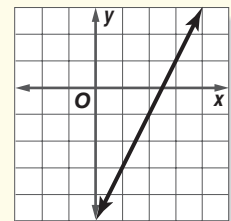
   (A) parallel to the line shown above, but shifted two units higher
   (B) parallel to the line shown above, but shifted two units lower
   (C) have a steeper slope, but intercept the *y*-axis at the same point
   (D) have a less steep slope, but intercept the *y*-axis at the same point

CONTENTS

Part 2 | Short Response/Grid In

**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**

10. What statement is needed in step 2 to complete this proof? **(Lesson 2-4)**

Given: $\dfrac{4x - 6}{3} = 10$

Prove: $x = 9$

Statements	Reasons
1. $\dfrac{4x - 6}{3} = 10$	1. Given
2. ___?___	2. Multiplication Property
3. $4x - 6 = 30$	3. Simplify.
4. $4x = 36$	4. Addition Property
5. $x = 9$	5. Division Property

The director of a high school marching band draws a diagram of a new formation as shown below. In the figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Use the figure for Questions 11 and 12.
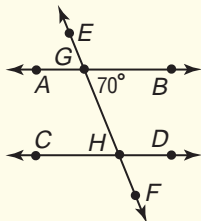
11. During the performance, a flag holder stands at point H, facing point F, and rotates right until she faces point C. What angle measure describes the flag holder's rotation? **(Lesson 3-2)**

12. Band members march along segment CH, turn left at point H, and continue to march along $\overline{HG}$. What is $m\angle CHG$ created by these marchers? **(Lesson 3-2)**

**13.** What is the slope of a line containing points $(3, 4)$ and $(9, 6)$? **(Lesson 3-3)**

Test-Taking Tip

Question 13 Many standardized tests provide a reference sheet that includes formulas you may use. Quickly review the sheet before you begin so that you know what formulas are available.

Part 3 | Open Ended

Record your answers on a sheet of paper. Show your work.

14. To get a player out who was running from third base to home, Kahlil threw the ball a distance of 120 feet, from second base toward home plate. Did the ball reach home plate? Explain and show your calculations to justify your answer. **(Lesson 1-3)**
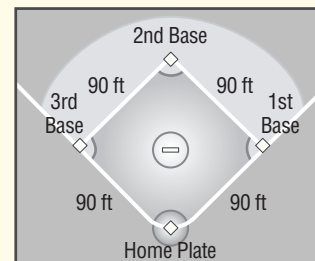
15. Brad's family has subscribed to cable television for 4 years, as shown below.
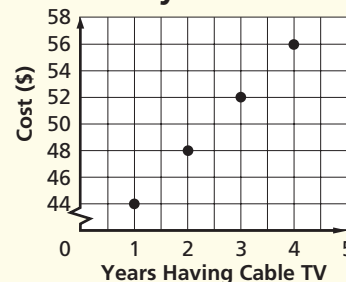
a. Find the slope of a line connecting the points on the graph. **(Lesson 3-4)**

b. Describe what the slope of the line represents. **(Lesson 3-4)**

c. If the trend continues, how much will the cable bill be in the tenth year? **(Lesson 3-4)**

CONTENTS